Е. И. Смирнов, С. А. Тихомиров, В. С. Абатурова

Феномен сложного знания в обучении математике как фактор формирования математической грамотности школьников

Введение. Эффективным направлением формирования математической грамотности школьников становится обучение математике на основе освоения обобщенных конструкторов сложного знания (например, современных достижений в науке) с весомым прикладным и математико-информационным потенциалом личностного развития. *Цель исследования:* разработать технологию формирования математической грамотности школьников в ходе освоения сложного математического знания и в контексте актуализации универсальных учебных действий средствами математического и компьютерного моделирования.

Материалы и методы. Материалы исследования основаны на историогенезе и актуализации процессов освоения сложного математического знания школьниками как эффективного механизма развития личности. Реализуются синергетический подход, средства цифровизации и методы наглядного моделирования для адаптации процессов освоения сложного знания к школьной математике с эффектом формирования математической грамотности школьников. Выбор и обоснование методов фундирования опыта личности создают эффект актуализации ядра универсальных учебных действий, проявляются в процессах индивидуализации деятельности школьников.

Результаты исследования. Впервые разработана технология формирования математической грамотности школьников на основе симбиоза математического и компьютерного моделирования в освоении математики сложного знания. Построены фундирующие кластеры и технология исследования и адаптации иерархий сложного разноуровневого знания (в том числе, современных достижений в науке) к школьной математике. Уточнены этапы и средства наглядного моделирования и фундирования опыта личности с эффектом формирования математической грамотности школьников в насыщенной информационно-образовательной среде.

Выводы. Образовательные практики показали высокую эффективность данной методики формирования математической грамотности школьников в процессе освоения современных достижений в науке. Такие дидактические решения и практики характеризуются способностью обеспечить в полной мере потребности каждого обучающегося в самообразовании и самоактуализации при освоении сложных знаниевых конструкторов и задают ценностный императив личностного развития, в том числе математической грамотности.

Ключевые слова: обучение математике сложного знания, симбиоз математического и компьютерного моделирования, практико-ориентированные задания, математическая грамотность школьников

Ссылка для цитирования:

Смирнов Е. И., Тихомиров С. А., Абатурова В. С. Феномен сложного знания в обучении математике как фактор формирования математической грамотности школьников // Перспективы науки и образования. 2021. № 6 (54). С. 285-299. doi: 10.32744/pse.2021.6.19

E. I. SMIRNOV, S. A. TIKHOMIROV, V. S. ABATUROVA

The phenomenon of complex knowledge in teaching mathematics as a factor of mathematical literacy forming of school students

Introduction. Mathematics teaching based on the development of complex knowledge generalized constructs (for example, modern achievements in science) becomes an effective direction for the formation of school student's mathematical literacy with a significant applied and mathematical-informational potential of personal development. The purpose of the study: to develop a technology for student's mathematical literacy formation during the development of complex mathematical knowledge and in the context of universal educational actions actualization by means of mathematical and computer modeling.

Materials and methods. The research materials are based on the historiogenesis and actualization of mastering processes of complex mathematical knowledge by students as an effective mechanism for personal development. A synergetic approach, digitalization tools and visual modeling methods are being implemented to adapt the mastering processes of complex knowledge to school mathematics with the effect of student's mathematical literacy forming. The choice and justification of methods for personal experience founding create the effect of core actualization of universal educational actions, manifest themselves in the processes of students' activities individualization.

The results of the study. For the first time, a technology for student's mathematical literacy formation based on the symbiosis of mathematical and computer modeling in mathematics development of complex knowledge has been developed. The founding clusters and research and adaptation technology of hierarchies of complex multi-level knowledge (including modern achievements in science) to school mathematics are constructed. The stages and means of visual modeling and personal experience founding with the effect of student's mathematical literacy forming in a rich information and educational environment are clarified.

Conclusion. Educational practices have shown the high efficiency of this method to school student's mathematical literacy forming in the process of modern achievements mastering in science. Such didactic solutions and practices are characterized by the ability to fully meet the needs of each school student in self-education and self-actualization when complex knowledge constructs mastering and set the value imperative of personal development, including mathematical literacy.

Keywords: teaching mathematics of complex knowledge, symbiosis of mathematical and computer modeling, practice-oriented tasks, school student's mathematical literacy

For Reference:

Smirnov, E. I., Tikhomirov, S. A., & Abaturova, V. S. (2021). The phenomenon of complex knowledge in teaching mathematics as a factor of mathematical literacy forming of school students.

Perspektivy nauki i obrazovaniya – Perspectives of Science and Education, 54 (6), 285-299. doi: 10.32744/pse.2021.6.19

Introduction

In the last decade, schools' digitalization has been declared the main trend of Russian education and is designed to provide the answers to the "explosive" emergence of new competencies, changes in the labor market and the openness of global information space. The international PISA test, a test that assesses the school student's functional literacy in different countries of the world and the ability to apply knowledge in practice (takes place every three years; the test involves teenagers aged 15 years), show that about a fifth of senior school graduates do not reach the threshold level of functional literacy (in each area – mathematical, natural science and reading) and about a third of school students in one of the areas. Russia ranks 27-35 in the world ranking. The main difficulties in performing tasks for monitoring of school student's functional (mathematical) literacy formation are identified:

- understanding the plot situation and translating it into the language of subject area, finding a solution method;
- working with information presented in various forms (figure, text, table, diagram);
- working with real data, quantities and units of measurement;
- interpretation of the result taking into account the proposed situation;
- the manifestation of independence, the use of educational and life experience.

Pedagogical experience, theory and practice, requests and challenges of real-life show that fundamental mathematical abilities play a central role in determining various success levels in mathematical literacy formation. In recent decades, scientists-mathematicians, philosophers, psychologists and teachers' efforts have methodologically revealed and theoretically proved that the following technological concept is able to show the mechanisms and the factors of fundamentality phenomenon actualization and the quality of mathematical education improving. It was school student's mathematical literacy formation (according to the identification and research of "problem areas" in mathematics development): self-organization and self-development of the individual on the basis of three spheres actualization of synergy manifestation of complex constructs for modern scientific knowledge (typology is identified and justified in [16]): *meaningful* (practice-oriented tasks, complex systems and real processes – fractals, chaos, nonlinear dynamics, cryptography, etc., mechanisms of self-organization and order), *procedural* (personal experience founding, cultures dialogue and communications, contexts, mathematical and computer modeling) and *personal-adaptive* (development of school student's creativity and criticality, visual modeling, development of the learning motivational sphere). Thus, mathematics teaching based on the development of complex knowledge generalized constructs (for example, modern achievements in science with significant applied and mathematical-informational potential) becomes an effective direction for school student's mathematical literacy formation (technology of modern achievements in science is created in [1; 2]).

This can be implemented in the course of step-by-step research and adaptation of complex knowledge generalized constructs to school mathematics with the included effect of practice-oriented tasks solving and the ability to interpret the tasks from real life: that is, to solve a wide range of tasks in various spheres of human activity, communication and social relations (practical methods for improving are realized in [24]). The integration concept of educational paradigms of complex knowledge mastering and the formation of

school student's mathematical literacy is the core actualization of universal educational actions that manifest themselves in the corresponding school student's cognitive activity (it is shown by the example of fractal geometry in [7]). This creates a precedent for expanding and deepening the personal experience based on its current state (it is necessary to take into account the school student's individual differences, forms, methods and means of complex knowledge mastering and the corresponding practice-oriented tasks should be multi-level), the formation and development of the learning motivational sphere (due to the actualization of samples and adaptation of modern, popular in life and accessible to perception scientific knowledge and technologies), the development of intellectual operations and abilities based on founding mechanisms. So, procedures of mathematical and visual modeling in mathematics development for complex constructs and school student's manifestation possibilities and functional, operational and instrumental competencies corrections are presented.

At the same time, it is necessary to implement the idea of not only developing, implementing and researching hierarchical and multi-level complexes of PISA-like tasks for school students, but also updating the basic generalized procedures and UEA (universal educational actions), integrating of mathematical knowledge and competencies. These can be: localization and structuring of information, understanding and generalization, integration and interpretation, modeling and reflection, self-assessment and self-control of knowledge that correlate with the levels and content of mathematical literacy in the context of the implementation of school student's research and play activities during the complex knowledge development. This integrative basis promotes the interaction, mutual influence, mutual enrichment of knowledge fields and will need to contribute to the formation of school student's functional (mathematical) literacy. The synergy of mathematical education in the context of the cultures dialogue and modern achievements in science adaptation (it is shown in the "aggravation mode" of S. P. Kurdyumov [12; 23]), whether it is inclusive education, distance learning or integrated courses, allows the creating conditions for improving the mathematical education quality, school student's educational and professional motivation with the disclosure of their individual characteristics («...turning yourself to culture and history ...» G. Hegel). At the same time, the post-non-classical thinking of modern individual, based on the nonlinearity of the surrounding reality, situationality and uncertainty in decision-making, multiple goal-setting and ambiguity of choice, strongly dictates the need and mastering possibility and accepting new scientific knowledge (as well as school student's mathematical literacy formation) by overcoming the complexity (for example, modern achievements in science), including this new knowledge, as an imperative of the transition from chaos to order. Especially such procedures are manifested in the study and complex mathematical knowledge adaptation to school mathematics by step-by-step and multifunctional manifestation of its generalized essence and its integration with school educational elements – such in our work are modern achievements in science (for example, fuzzy-logic or fuzzy set theory [24]).

Thus, even the school student's mathematical literacy in the process of research activity can act as one of the important attractors of symbiosis phased deployment in successive iterations of generalized procedures study (universal educational actions) and complex knowledge adapting processes of basic educational elements development to school mathematics. This dictates the need to build, investigate and consider a multi-level complex as a condition for building the parameters of the order of mathematics motivated mastering and the transition to dynamically stable states of a new level of techniques and

acts complexity of mathematical thinking. Such development of complex mathematical knowledge by school students allows the creating of research situations leading to the ability to maintain the dynamic stability of mental activity state (formation of mathematical literacy in research activities course). It is possible with acceptable values of internal or external disturbances (fluctuations) of mathematical activity in study processes of generalized constructs adaptation of modern achievements in science.

Objective: to develop the methodological, theoretical and technological foundations for the creation and functioning of rich information and educational environment for mathematics teaching and supporting the design and school student's research activities in general education structure based on the development of complex knowledge generalized constructs:

- to develop and justify the technology of organizing and supporting the design and school student's research activities and the manifestation of mathematical education synergy based on the development of complex knowledge generalized constructs (for example, modern achievements in science adaptation) in rich information and educational environment;
- to scientifically substantiate and develop a didactic model for supporting the school student's project and research activities in rich information and educational environment based on personal-activity and synergetic approaches, mathematical and computer modeling symbiosis and self-organization of school student's cognitive activity.

The leading idea is as follows: the key aspect of the phenomenon of school student's mathematical literacy formation and synergetic effects manifestation in mathematics teaching of complex knowledge based on modern achievements in science adaptation is the possibility of updating generalized stages and studying the characteristics of essence mastering of complex mathematical knowledge, phenomena and procedures, creating conditions for communication and cultures dialogue, identifying attributes of self-organization of content, processes and interactions (attractors, bifurcation points, pools of attraction, iterative procedures, etc.) in the course of "problem areas" mathematics mastering.

Review of sources

Since ancient times, philosophers, mathematicians, teachers have been wondering about the complexity of sciences, tasks, texts, systems, processes and phenomena. Development of complexity is mediated by extensive experimental material, practice and the interdependence of integrative processes in science, technology, economics, social transformations and educational paradigms (philosophical concept and historiogenesis are presented by V.G. Budanov [12]). Polyvalence, multiplicity, multipolarity, unpredictability, emergence and disequilibrium of modern world cannot but be linked to the categories of the essence development of objects, phenomena and processes through the manifestation of transitions patterns to higher levels of the complexity as components of a specifically universal theory of development. Teachers and psychologists note in complexity phenomenon the quantitative characteristics of data, conditions, possibilities of text transformation, the volume of logical connections, the degree of studied issues abstraction (psychological interpretation by S. L. Rubinstein is presented in [27]). The ancient philosophers Plato, Aristotle, and Stagirite established an ontological distinction between a simple and

complex, which is expressed in pairs of opposites traditional for ancient Greek thought, such as "one-many", "elementary-composite", "necessary-accidental". Solving problems of computational complexity (theoretical analysis and the results are considered in [25; 26]) showed that it is the time characteristics that play the most important role in assessing the complexity of the problem (P-class problems (P-difficulty) – polynomial time, the traveling salesman problem – exponential time, etc.). "Complexity means many different things — there is descriptive complexity and computational complexity. The algorithm can be extremely complex in terms of the way it is constructed and at the same time work very quickly, since its computational complexity is low. Thus, we have different concepts of complexity" (classification of R.M. Karp is presented in [28]). Algorithmic complexity (Kolmogorov complexity is an internal characteristic of finite object, equal to the length of shortest binary code by which a universal algorithm can restore this object. Algorithmic information theory by A. N. Kolmogorov was an attempt to extend the information theory of K. Shannon to the non-stochastic case (including the phenomenon of information entropy concept [26])). There are approaches when complexity is associated with formation time of system or with its hierarchical structure, as well as with the formation probability of system from the initial elements, sometimes complexity can mean the ability of system to generate the semiotic information links and to interact with the external environment on their basis, allowing to implement a hierarchical management structure. The concept of "complexity is the occurrence of bifurcation transitions away from equilibrium and in the presence of suitable nonlinearities, symmetry breaking above the bifurcation point, as well as the formation and support of macroscopic scale correlations" (following I. Prigogin [23]). Considering scientific knowledge as a complex system of generalized constructs variety in cognition unity, we note the didactic aspect of its adaptation to school mathematics by expanding the essential and phenomenological characteristics of reflecting current applications and the need for computer and mathematical modeling in ontogenesis. There was a need to develop an environment for distance learning in mathematical disciplines as part of methodological initiatives deployment of mathematics developers-teachers, as well as complexes of online courses and remote environments; it is necessary to develop the provision of ICT support tools (including the mathematical package of computer algebra Mathematica) in solving the complex problems in mathematics teaching [8-9]; the "tetrad" technology will be developed in school student's research activities: the peculiarity here is that school students will have to perform four types of creative activities: a) creative mathematical activity; b) construction of fractal sets with the development of algorithms and high-level programming languages; c) performing laboratory work in mathematics (with computer experiments [13; 14]); d) studying of scientists creative biographies and creating the artistic compositions using fractals and ICT.

Methodology and Methods

Complex knowledge arises in complex systems and generates multiple complex tasks. The historical experience of world problems solving in mathematics shows that, for example, the result of following problems knowing is a complex mathematical knowledge: the problem of 4 colors for coloring maps (V. Haken, K. Appel); Riemann hypothesis about zeros of ζ - function; binary Goldbach problem; transcendence of the numbers π and e ; rationality of Euler-Mascheroni number; problem of P and NP problem for the computational efficiency

of iterative problems (P. Cook, L. Levin, A. Wigderson); Fermat's Great Theorem (A. Wiles); fractal characteristics of Schwartz cylinder (T. Schwartz, B. Mandelbrot, E. I. Smirnov, etc.). We want to use the generalized construct of complex mathematical knowledge for mathematical literacy forming so it can represent the applied or practice-oriented knowledge, the learning and essence manifestation of which is based on mathematical and computer modeling symbiosis. These can be fractal geometry elements: variations of Julia and Mandelbrot sets [2; 7], the game of "chaos" in the randomized construction and study of "Serpinsky napkin" fractal characteristics and its generalizations [5; 6; 10], the study of strange attractors of Henon, Roesler and Lorentz [2; 16]; graph theory (transport networks, queuing theory [28]); fuzzy sets and fuzzy logic (L. Zadeh, Ye. Mamdani [24]); information encoding and encryption (K. Shannon, Huffman [16]); stochastic methods of optimization problems (J. Holland, J. Koza [28]), etc.

The study of synergy manifestation processes of complex knowledge in mathematics teaching turns out to be directly and naturally related to forming effect not only the functional (mathematical) literacy, but also the category of computational thinking. The fact is that the basis for solving and researching practice-oriented problems is mathematical modeling, i.e., substitution of real, materialized and ideal objects and procedures with sign-symbolic, geometric, relational, procedural, frame, fractal models as generalized constructs embedded in the structural and logical field of mathematical signs and symbols that obey myriad laws and laws that have the status of absolute truths. However, the implementation of logical inference does not always lead to an exact result (this is also the theorem of K. Godel's incompleteness, the multiplicity of syllogisms, which sometimes becomes unrealistic, the need for computational procedures, and much more), so that, especially when studying of complex knowledge, information technologies and computational procedures are required, at least to obtain an approximate solution. In other words, we need the so-called computational thinking, when following Zh. Wing "... thought processes involved in problems formulation and their solutions in such a way that solutions are presented in a form that can be effectively implemented using information processing tools" [25]. E. K. Henner [26] gives a number of definitions examples of computational thinking (CT), which directly arises when operating with complex knowledge and make an impact on mathematical literacy formation. Below are some of these judgments:

- CT is closely related to procedural thinking, the definition of which was formulated by Seymour Papert back in 1981 [11]. Procedural thinking includes the development, presentation, testing and debugging of procedures, which are a set of step-by-step instructions, each of which can be formally interpreted and executed by a special performer, such as a computer or automatic equipment;
- CT is associated with the study of the intelligence mechanisms, accompanied by practical applications, expressed in human intelligence strengthening through the tools using that help to automate the solution of complex tasks;
- CT-a way to formulate the accurate methods for effective problem solving, including a thorough analysis of problems and solution procedures.

All the results obtained characterize the synergy manifestation of complex knowledge in mathematical education at school based on modern achievements in science adaptation, mainly in the forms of integrative and elective courses implementation, project activities and web quests, laboratory-calculation and resource classes, including in gaming activities.

Thus, **the study object and our approach** are based on student's learning of complex knowledge (modern achievement in science) with actualization of *generalized universal*

educational actions (GUEA) and mathematical literacy forming. *Complex knowledge in our approach* is the result of knowledge about the content and semiotic information connections of nonlinear systems, objects and phenomena of real and virtual world, presented in the unity of descriptive and computational diversity and hierarchies of content representation with the possibility of updating bifurcation transitions and various interpretations, and generations of essence forms manifestation.

Methods. Mathematical modeling is one of the most effective activity to define the essence and adaptation the steps of student's complex knowledge learning (methodological aspects of teaching geometry based on digital educational resources are presented in [13]). The conceptual, natural science, mathematical modeling are important levels of essence revealing of practice-oriented tasks solving (improving the methods of pedagogical diagnosis are realized in [9]). Combination of visual modeling (computer design of nonlinear growth of "areas" of an irregular Schwarz cylinder is developed in [5]) and personal's experience founding methods (created and realized in [6]) define the opportunities to actualize the student's individual educational trajectories. Computer modeling is actualized and applied for investigation of "problem zones" during complex knowledge solving and actualization of relevant practice-oriented subtasks (elements of the theory of discrete dynamical systems is presented in [2; 8]). This leads to the identification and revealing of following characteristics of complex knowledge about nonlinear systems, objects and phenomena of real and virtual world:

- the possibility of content components interpreting and generating and semiotic information links with pronounced applied effects;
- information about the unity of descriptive and computational diversity and hierarchies of content representation and semiotic information links;
- the possibility of updating bifurcation transitions and various interpretations, and generation of essence forms manifestation by methods of mathematical and computer modeling.

Results

In our practice technology, we are talking about identifying generalized constructs and procedures in information processes that accompany the study of complex knowledge and mathematical literacy forming.

1. Thus we are interested in generalized constructs and procedures for solving and researching the complex knowledge based on mathematical and computer modeling (including gaming activities) with the actualization of school student's mathematical literacy during practice-oriented procedures for solving PISA-like tasks. An effective construct, method and mechanism for the formation of school student's mathematical literacy may be the deployment of following synergy manifestation stages of complex knowledge learning in mathematical education at school. It may be: motivational (self-actualization ("I'm interested in this")); approximate information saturation (self-determination ("what can I do")); process-activity (self-organization ("I'm able to manage the process")); control-correctional (evaluation of empirical verification of results); generalizing-transforming (self-development of the individual ("I can do something new")); at the same time, it is necessary to develop the methods for the selection, justification and development of psycho diagnostic methods and evaluation procedures for identifying of teacher's professional deficits and

technologies for identifying of synergetic effects in teaching mathematics (evaluation of synergetic effects of integration of knowledge and active learning in higher school are developed in [3; 4]).

2. The category of personal abilities associated with organizing functional system and performing actions, making decisions, evaluating of an action result was studied in the works of such scientists as S. L. Rubinstein [27]. Following the classical analysis of V. D. Shadrikov's abilities [18], we define school student's *mathematical (functional) literacy* as a socially approved of an individual's functional systems properties severity measure, manifested in the success of mathematical activities implementation in sciences and real-life development. Social approval refers to compliance with the regulatory documents of various state institutions in the field of educational policy: requirements of the second-generation Federal State Educational Standard, the State Program of Russian Federation "Development of Education" (2018-2025), the Decree of the President of Russian Federation and the Government Resolution of 2013 "On the Concept of Mathematical Education Development in Russian Federation", etc. The success of mathematical activity mastering and implementing is understood (in accordance with the requirements of the PISA (Program for International School Student Assessment)) as an individual's ability to formulate, apply and interpret mathematics in context variety, to express well-founded judgments and make decisions. PISA recommends the mathematics mastering in 4 areas: measurement and relation, space and form, quantity and uncertainty ([20]). The excessive coverage of these areas in accordance with Russian mathematical education traditions is determined by seven meaningful lines of school mathematics: numerical, functional, geometric, identical transformations, equations and inequalities, stochastic and algorithmic. Therefore, the components of school student's mathematical literacy content should be determined by the need to reflect these seven content lines according to niches: to know, to be able, to possess each of which (in accordance with the requirements of PISA) is differentiated by three levels: threshold, basic and advanced (complex)). However, since the key question for us is the need to form the *generalized universal educational actions (GUEA)*, these seven content lines are integrated in interaction in the manifestations of the following GUEA: localization and structuring of information; understanding; integration and interpretation; reflection; modeling; self-assessment and self-control. Each of GUEA's components is further disclosed in the characteristics, meters and complexes of multi-level practice-oriented tasks that have a complex multi-stage nature of the solution, research and used mathematical and information methods, as well as mathematical and computer modeling tools.

3. It is their actualization, as S. L. Rubinstein points out, that is the basis for abilities formation, including mathematical literacy [27]. The following is a model of mathematical literacy formation and diagnosis and formation and diagnosis stages of school student's mathematical literacy (Fig. 1).

4. Thus, mathematics teaching based on the development of complex knowledge generalized constructs (for example, modern achievements in science with significant applied and mathematical-informational potential) becomes an effective direction for school student's mathematical literacy formation. At the same time, the task is to create a rich information and educational environment for mathematics teaching by changing the content of educational programs in the direction of complex knowledge generalized constructs mastering and remote environments supporting, a symbiosis of mathematical and computer modeling. This is realized in the course of step-by-step research and the adaptation of complex knowledge generalized constructs to school mathematics with the

Figure 1 Structural and functional model of the formation and diagnosis of mathematical literacy (ML)

included effects of practice-oriented tasks solving and the ability to interpret the tasks from real life. So, it is necessary to solve a wide range of tasks in various spheres of human activity, communication and social relations. *The concept of educational paradigms integration by complex knowledge mastering and school student's mathematical literacy formation is the actualization of universal educational actions core that manifest themselves in the corresponding school student's cognitive activity.* This creates a precedent for expanding and deepening the personal experience based on its current state (it is necessary to take into account the school student's individual differences, forms, methods and means of complex knowledge mastering and the corresponding practice-oriented tasks should be multi-level). It lead to formation and development of learning motivational sphere (due to the actualization of modern samples and adaptation, popular in life and accessible to scientific knowledge and technologies perception), the development of intellectual operations and abilities based on founding mechanisms, mathematical and visual modeling of manifestation possibilities and school student's functional, operational and instrumental competencies correction in the development of mathematics complex constructs and procedures. Thus, the process implementation of functional literacy quality improving in the development of school mathematics is now possible on the basis of synergetic principles and approaches actualization in the context of modern achievements in science adapting to school mathematics. Such educational systems are characterized by the ability to fully meet the needs of each school student in self-education and self-actualization when complex knowledge constructs mastering and set the value of personal development imperative. Therefore, a dialogue of information, humanitarian, mathematical and natural science cultures is also necessary in the development of complex knowledge mathematics, which is associated with the solution and research of practice-oriented PISA-like tasks, activates the synergy mechanisms. So, it is become a factor of personal's self-organization and the links in universal educational actions formation and integral structures formation in mathematics teaching at school.

5. At the same time, it is necessary to implement the idea not only developing, implementing and researching of hierarchical multi-level complexes of PISA-like tasks for school students [15], but also *updating the basic generalized procedures and ULA (universal educational actions), mathematical knowledge and competencies integrating. These can be: localization and structuring of information, understanding and generalization, integration and interpretation, modeling and reflection, self-assessment and self-control of knowledge that correlate with mathematical literacy levels and content in the context of research implementation and school student's play activities during the complex knowledge development.* This integrative basis promotes the interaction, mutual influence, mutual enrichment of areas knowledge and will need to contribute to school student's functional (mathematical) literacy formation [17]. The mathematical education synergy in the context of cultures dialogue and modern achievements in science adaptation (chess instruction improve mathematical problem-solving ability is presented in [21]), whether it is inclusive (inclusive) education, distance learning or integrated courses, allows the creating conditions for improving the mathematical education quality, school student's educational and professional motivation with their individual characteristics disclosure.

At the same time, the procedures for generalized essence mastering of complex knowledge and the transition in the individualization processes in school student's nearest development zones will be more pronounced and directed if the indicative and informational foundations of school student's research activities design are cemented

by a specially designed founding cluster. It really manages and observe the research levels and the essence manifestations of complex knowledge generalized construct [19]. Thus, school student's experience founding as an innovative mechanism for personal development and comprehension of the essence of complex knowledge generalized construct (in the course of modern achievements in science mastering) can unfold in three educational niches: the content of mathematics teaching at school, the technology of implementing adaptive processes and the development of school student's personal qualities (presented and realized in [22]).

It is the educational processes management based on complex knowledge development by mathematical and computer modeling means. It can give a powerful motivational charge to mathematical disciplines study. As a result, the interest in mathematics with real development of theoretical and empirical thinking (comparison, analogy, analysis, synthesis, etc.) will increase and school student's mathematical literacy level, creativity and critical thinking will increase. At the same time, the possibility of modern achievements in science adapting to school mathematics and computer interactive interaction with the educational subject enhances the developing effect and increases the educational motivation, reveals connections with real life and practice, creates the synergetic effects phenomenon in complex mathematical knowledge development.

Discussion

So, the task for the coming years is not only to achieve a stable threshold level in PISA testing, at which school students begin to demonstrate the knowledge application and skills in the simplest extracurricular situations, but also to achieve the ability to solve the complex problems. Priority is given to situations when the school student's ability to use the existing knowledge and skills, to obtain the new information is manifested, but creative school students who are independently thinking and able to function in difficult conditions and to master of complex knowledge are required. The feature of this paper (what is consistent with the authors' view (theory of self-organization and system logical approach to methodical disclosure are presented in [16; 17]) is to form the mathematical literacy by using complex mental operations. We agree with researchers that such technology creates a precedent for expanding and deepening the person's experience based on his current state (psychology of cognition and methodological aspects of teaching geometry based on digital educational resources are developed in [11; 13]). It is necessary to take into account the school student's individual differences, i.e., practice-oriented tasks should be multi-level, the formation and development of the motivational sphere of teaching, have a strong links and have been stimulated the phenomenon of complexity actualization. Due to the actualization of modern samples and adaptation, popular in life and accessible to perception, scientific knowledge and technologies we support the development of intellectual operations and abilities based on founding mechanisms (what does not always agree with the researcher's opinion (innovative pedagogical practices in higher education are realized in [22])). Actualization of mathematical and visual modeling symbiosis in managing process of manifestation possibilities and school student's functional, operational and instrumental competencies make the basis for correction in the development of mathematics complex constructs and procedures.

Conclusion

Thus, an effective direction for school student's mathematical literacy formation can be mathematics teaching on the basis of complex knowledge mastering. It turned out that necessary to build the hierarchies of complex multi-level knowledge, methods and tools in cognitive activity, relying on didactic rules and patterns of mathematical activity mastering based on a synergetic approach (fractal geometry, fuzzy sets and fuzzy-logic, chaos and catastrophe theory, stability of dynamic systems and nonlinear dynamics, the theory of encoding and encryption of information, etc.). At the same time, new technology create a rich information and educational environment for mathematics teaching and mathematical literacy formation by changing the educational programs content in the direction of complex knowledge generalized constructs mastering and organizing support for remote environments and computer modeling. This is realized in the course of step-by-step research and complex knowledge adaptation with the context of practice-oriented subtasks solving, with the ability to effectively interpret the situations from real life. It is making to solve a wide range of tasks in various spheres of human activity, communication and social relations. Thus, we demonstrate that the process implementation of functional literacy quality improving in the development of school mathematics is now possible on the basis of synergetic principles and approaches actualization in the context of modern achievements in science adapting to school mathematics. Such educational systems are characterized by the ability to fully meet the needs of each school student in self-education and self-actualization when complex knowledge constructs mastering and set the imperative of personal development value. Therefore, a dialogue of information, humanitarian, mathematical and natural science cultures is also necessary in the development of mathematics complex knowledge, which activates the synergy mechanisms and is a factor of self-organization and links in the integral structures' formation of mathematics teaching at school. Thus, mathematics teaching at school should take place in an information-rich educational environment of complex level knowledge mastering in the context of mathematical, informational, humanitarian and natural science dialogue cultures and teachers and school student's didactic efforts integration. It should be developed in the direction of the essences revealing of basic educational elements (concepts, theorems, procedures, algorithms, ideas) as a fundamental phenomenon of education.

Acknowledgements

The work was carried out within the framework of the state task of the Ministry of Education of the Russian Federation No. 073-00077-21-02 for carrying out scientific research on the topic "The mechanism of scientific and methodological support of teachers on the formation of school student's functional literacy: transfer of educational technologies" (registry entry no.730000F.99.1.BV10AA00006).

REFERENCES

1. Smirnov E.I., Bogun V.V., Uvarov A.D. Synergy of mathematical education: An introduction to analysis. Yaroslavl, Chancellor Publ., 2016. 216 p. (in Russ.)
2. Sekovanov V.S. Elements of the theory of discrete dynamical systems. Saint-Petersburg, Lan' Publ., 2016. 180 p. (in Russ.)
3. Dvoryatkina S.N., Smirnov E.I. Evaluation of synergetic effects of integration of knowledge and activity based on computer modeling. *Modern information technologies and IT education*. Moscow, MSU Publ., 2016. pp. 35-42. (in Russ.)
4. Verbitsky A.A. Active learning in higher school: a contextual approach. Moscow, Higher School Publ., 1991. 207 p. (in Russ.)
5. Smirnov E.I., Uvarov A.D., Smirnov N.E. Computer design of nonlinear growth of "areas" of an irregular Schwarz cylinder. *Eurasian Scientific Review*, 2017, vol. 30, no. 8, pp. 35-55. (in Russ.)
6. Ostashkov V.N., Smirnov E.I., Belonogova E.A. Education Synergy in Research of Attractors and Basins of Nonlinear Mapping Attraction. *Yaroslavl Pedagogical Bulletin*, 2016, no. 6, pp. 146-157. (in Russ.)
7. Mandelbrot B.B. Fractal geometry of nature. Moscow, Institute of Computer Research, 2002. 656 p. (in Russ.)
8. Zykova T.V., Kuznetsova I.V. Synergy of Students ' Network Interaction in the Course of Mathematical Knowledge Development. *Yaroslavl Pedagogical Bulletin*, 2017, no. 5, pp. 95-102. (in Russ.)
9. Dvoryatkina S.N., Masina O.N., Shcherbatykh S.V. Improving the Methods of Pedagogical Diagnosis and the Control of Mathematical Knowledge Based on the Modern Achievements in Science. *Educational Psychology in Polycultural Space*, 2017, no. 1 (37), pp. 71-77. (in Russ.)
10. Barnsley M., Igudesman K.B. Overlapping iterated function systems on a segment. *Russian Mathematics*, 2012, no. 12, pp. 1-12. (in Russ.)
11. Bruner J. Psychology of cognition. Beyond the immediate information. Moscow, Direct-Media Publ., 2008. 482 p.
12. Budanov V.G., Eskov V.N. Postnonclassics and the third paradigm of natural science. *Complexity. The mind. Post-non-classical*, 2019, no. 1, pp. 53-61. doi: 10.12737/article_5cb82a52055b19.67238854.
13. Dalinger V.A. Methodological aspects of teaching geometry based on digital educational resources. *Continuum. Mathematics. Computer science. Education*, 2020, no. 1 (17), pp. 10-15. doi: 10.24888/2500-1957-2020-17-1-10-15. (in Russ.)
14. Dvoryatkina S.N., Karapetyan V.S., Rozanova S.A. The multiplicity of goal-setting in pedagogical activity: mathematics on a chessboard. *Higher education in Russia*, 2019, vol. 28, no. 4, pp. 81-92. doi: 10.31992/0869-3617-2019-28-4-81-92. (in Russ.)
15. Malinetsky G.G., Sirenko S.N. Reflection of PISA research education in the post-soviet space. *Bulletin of the Moscow University. Series 20: Pedagogical Education*, 2020, no. 1, pp. 35-69. (in Russ.)
16. Malinetsky G.G. Ideology in the context of natural sciences and the theory of self-organization. *Strategic priorities*, 2020, no. 1-2 (25-26), pp. 93-109. (in Russ.)
17. Monakhov V.M., Tikhomirov S.A. A Systemological Approach to Methodical Disclosure of the Second Generation FSES Prognostic Potential. *Yaroslavl Pedagogical Bulletin*, 2016, no. 6, pp. 117-126. (in Russ.)
18. Shadrikov V.D. Systemogenesis of activity. Game. Teaching. Work. Vol. 1. System genesis of professional and educational activities. Yaroslavl, Yaroslavl State University named after P. G. Demidov, 2017. 326 p. (in Russ.)
19. Burgoyne A.P., Sala G., Gobet F., Macnamara B., Campitelli G., Hambrick D. The relationship between cognitive ability and chess skill: A comprehensive meta-analysis. *Intelligence*, 2016, vol. 59, pp. 72-83. doi: 10.1016/j.intell.2016.08.002.
20. PISA 2022 Mathematics Framework. Available at: <https://pisa2022-maths.oecd.org/#Twenty-First-Century-Skills> (accessed 31 August 2021).
21. Sala G., Gobet F. Does chess instruction improve mathematical problem-solving ability? Two experimental studies with an active control group. *Learning and Behavior*, 2017, vol. 45, no. 4, pp. 414-421. doi: 10.3758/s13420-017-0280-3.
22. Santos J., Figueiredo A.S., Vieira M. Innovative Pedagogical Practices in Higher Education: An Integrative Literature Review. *Nurse Education Today*, 2019, vol. 72, pp. 12-17. doi: 10.1016/j.nedt.2018.10.003.
23. Nikolis G., Prigogine I. Exploring Complexity. Moscow, URSS, 2017. 360 p.
24. Chaira T. Fuzzy Set and Its Extension: The Intuitionistic Fuzzy Set. Wiley & Sons Ltd., 2019. 304 p. doi: 10.1002/9781119544203.
25. Wing J. Computational Thinking. *Communications of the ACM*, 2006, vol. 49, no. 3, pp. 33-35.
26. Henner E.K. Computational thinking. *Education and Science*, 2016, no. 2 (131), pp. 18-31.
27. Rubinstein S.L. Fundamentals of general psychology. Saint-Petersburg, Peter Publ., 2015. 713 p.
28. Karp R.M. Combinatorics, complexity, randomness: lectures of the Turing Prize laureates. Moscow, Mir Publ., 1993. pp. 498-521.

Информация об авторах

Смирнов Евгений Иванович

(Российская Федерация, Ярославль)
Доктор педагогических наук, профессор,
заведующий кафедрой математического анализа,
теории и методики обучения математики
Ярославский государственный педагогический
университет им. К. Д. Ушинского
E-mail: smiei@mail.ru
ORCID ID: 0000-0002-8780-7186
Scopus Author ID: 55734369900

Тихомиров Сергей Александрович

(Российская Федерация, Ярославль)
Кандидат физико-математических наук, доцент
кафедры геометрии и алгебры
Ярославский государственный педагогический
университет им. К. Д. Ушинского
E-mail: satikhomirov@mail.ru
ORCID ID: 0000-0002-7409-8464
Scopus Author ID: 9133507700

Абатурова Вера Сергеевна

(Российская Федерация, Владикавказ)
Кандидат педагогических наук, старший научный
сотрудник отдела функционального анализа
Южный математический институт (филиал)
Владикавказский научный центр
Российской академии наук
E-mail: veronika-abaturova@yandex.ru
ORCID ID: 0000-0002-8780-7186
Scopus Author ID: 55734369900

Information about the authors

Eugeny I. Smirnov

(Russian Federation, Yaroslavl)
Doctor of Pedagogical Sciences, Professor,
Head of Mathematical Analysis, Theory and Methods of
Teaching Mathematics Department
Yaroslavl State Pedagogical University named after K. D.
Ushinsky
E-mail: smiei@mail.ru
ORCID ID: 0000-0002-8780-7186
Scopus Author ID: 55734369900

Sergey A. Tikhomirov

(Russian Federation, Yaroslavl)
PhD in Physical and Mathematical Sciences,
Associate Professor of Geometry and Algebra Department
Yaroslavl State Pedagogical University named after K. D.
Ushinsky
E-mail: satikhomirov@mail.ru
ORCID ID: 0000-0002-7409-8464
Scopus Author ID: 9133507700

Vera S. Abaturova

(Russian Federation, Vladikavkaz)
PhD in Pedagogical Sciences,
Senior Researcher of Functional Analysis Department
Southern Mathematical Institute (branch)
Vladikavkaz Scientific Centre of the Russian Academy of
Sciences
E-mail: veronika-abaturova@yandex.ru
ORCID ID: 0000-0002-8780-7186
Scopus Author ID: 55734369900