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Кейс-тесты выявления профессиональных дефицитов педагогов в интерпретации сложного знания

Введение. Формирование функциональной (математической) грамотности школьников является в последние годы базовым трендом математического образования в основной школе в связи с результатами международного тестирования PISA. Педагогический опыт, теория и практика, запросы и вызовы реальной жизни показывают, что центральную роль для педагога в определении различных уровней успешности формирования математической грамотности играют фундаментальные математические способности школьников и ситуации преодоления трудностей в освоении сложного знания.

Цель исследования – выявить предметные и методические дефициты педагогов в интерпретации сложного знания как фактора формирования математической грамотности школьников.

Материалы и методы. Материалы: диагностические кейс-тесты; онлайн-сервис Google Forms. Методы описательной статистики: среднее арифметическое, стандартное отклонение и др.

Результаты. Выявлено, что большинство педагогов (80%) адекватно оценивает свой научно-методический потенциал, толерантны к инновациям в дидактике. Низкие результаты педагогов (43,7%) свидетельствуют о наличии предметных дефицитов у учителей математики в области применения математических знаний в решении практико-ориентированных заданий. Средние показатели состояния методических дефицитов педагогов (62,3%) свидетельствуют о готовности педагога к профессиональному саморазвитию, возможности адаптировать и использовать знание современных проблем науки при решении практико-ориентированных заданий с эффектами мотивации и самоорганизации обучающихся.

Заключение. Предметные и методические компетенции педагога должны охватывать содержание и способы адаптации сложного знания (современных достижений в науке) к школьной математике, вариативность форм и средств представления обобщенных конструктивов, владение методами математического и компьютерного моделирования, знание образцов и эталонов востребованных приложений сложного знания к реальной жизни, технологиям и производствам. Практико-ориентированные задания возникают в обучении математике при этом как компонент проектной и исследовательской деятельности.

Ключевые слова: математическое образование, сложное знание, профессиональные дефициты педагогов, математическая грамотность

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Case tests as a tool for identifying of teacher’s professional deficits in the interpretation of complex knowledge

**Introduction.** The formation of functional (mathematical) literacy of students has been the basic trend of mathematical education in secondary school in recent years in connection with the results of the international PISA testing. Pedagogical experience, theory and practice, requests and challenges of real life show that fundamental mathematical abilities and situations of overcoming difficulties in mastering complex knowledge play a central role in various levels determining in the formation of mathematical literacy success. The purpose of the study is to identify the subject and methodological deficiencies of teachers in the interpretation of complex knowledge as a factor in the formation of mathematical literacy of students, using the original competence meters in case tests e form and using Google Forms.

**Materials and methods.** Materials: diagnostic case-tests; online service Google Forms. Methods of descriptive statistics: arithmetic mean, standard deviation, etc.

**Results.** It was revealed that teacher’s majority of (80%) adequately assesses their scientific and methodological potential, are tolerant of innovations in didactics. Low average results of teachers (43.7%) are shown, indicating the presence of subject deficits in mathematics teachers in the field of applying mathematical knowledge in solving practice-oriented tasks. The average results of teachers’ methodological deficits (62.3%) indicate the teacher’s readiness for professional self-development, the ability to adapt and knowledge using of science’s modern problems in solving practice-oriented tasks with student’s motivation and self-organization effects.

**Conclusions.** The subject and methodological competencies of a teacher should cover the content and ways of complex knowledge adapting (modern achievements in science) to school mathematics, the variability of forms and means of generalized constructs presenting, knowledge of mathematical and computer modeling methods, knowledge of samples and standards of popular applications of complex knowledge to real life, technologies and industries. Practice-oriented tasks arise in teaching mathematics at the same time as a component of student’s project and research activities.

**Keywords:** mathematical education, complex knowledge, professional deficits of teachers, mathematical literacy

**For Reference:**
With its 2021 report “Reimagining our futures together: A new social contract for education”, the International Commission on the Futures of Education chaired by Sahle-Work Zewde issued a call for collective action to transform education so that knowledge and learning best support the inter-generational work of building just, peaceful and sustainable futures for humanity and the living planet Earth. Education will have to serve us very differently than it has in the past. But this transformation also must be accomplished in a different way than before. We will need strengthened public dialogue and more inclusive participation – with priority placed on those whose voices, cultures, and knowledge’s have been most excluded and marginalized.

Mathematical education in Russia in the modern period is at the point of bifurcation of significant changes both in the content of mathematics teaching, and integration methods and technologies of mathematical and computer modeling within the general trend of education digitalization. The needs of society in digital and mathematical literacy of individuals, demands of digital economy are realized in expanding the base of STEAM education and requests for STEAM specialties. Currently STEAM (Science, Technology, Engineering, Art and Mathematics) is considered the strongest combination of academic areas and teaching methods for society education and innovative development. The overview [17] shows that digital literacy and computational thinking should become as integral competencies obtained in system education, and within the framework of university programs, emphasis should be placed on training specialists who are able to work with various automation technologies. The effectiveness of modern society functioning and the implementation of high technologies in production require the creative and critical individuals’ education who study in conditions of their scientific potential self-development during the development of complex systems and knowledge, especially in the development of school mathematics. Complex knowledge is a symbiosis of empirical and theoretical as a cognition result. At the same time, complex knowledge in the process of its cognition generates a symbiosis of mathematical and computer modeling, which is inherent in modern achievements manifestations in science and finds the effective applications in real life and practice, technologies and production (fractal geometry, fuzzy sets and fuzzy logic, neural networks and artificial intelligence, theory of encoding and encryption of information, cellular automata, robotics, virtual and augmented reality, non-Euclidean geometries, Schwarz cylinder and cone, etc.). It dictates the need for science and education integration as a fundamental paradigm for the development of school mathematical education. In recent decades, a number of world leading scientists have drawn the attention to need of educational paradigms revising in the direction of synergetic paradigm implementing. It concern with complex systems and knowledge mastering or self-organization paradigm in education (H. Haken [22], G.G. Malinetsky [4], B. Mandelbrot [5] et al.). A schoolboy and a student should get acquainted with the nonlinear, including synergetic style of thinking in post-non-classical sciences, know and find the associations in real life of such phenomena of collective ordering as the Jabotinsky-Belousov effect, Benard cells ("the road of giants"
in Ireland), the Ginzburg-Landau theory of superconductivity in the quantum system, the Lotka equations – Volterra in the predator-prey system, the Koch snowflake and Schwartz cylinder, Ferhulst scenario and the “butterfly effect” of the strange Lorentz attractor, etc. (A.I. Kreyk [8], V.N. Ostashkov [28], S.N. Dvoryatkina [7]). It is these and similar directions that provide a unique opportunity for student’s motivated involvement in coordinated development process of subject content in an open and saturated information and educational environment. It should lead to prediction and self-organization of cognitive activity, assessment and dynamics of personal changes, development and self-organization of supra-situational activity and visual modeling both in the process of formal and informal mathematical education. At the same time, a modern mathematics teacher should be ready for such systemic changes in professional activity and possibility of updating students' project and research activities based on mathematical and computer modeling symbiosis during of complex knowledge development, to educate the need for teamwork, the natural need for cultures dialogue and interests’ diversification, while forming students' mathematical literacy. This leads to the need to identify the content of professional deficits of teachers in project and research activities effective management of each student in complex systems development and knowledge: creating conditions for multiple goal-setting, the presence of a rich information and educational environment, deployment of hierarchical bases and complexes of multi-stage mathematical and informational research tasks with an availability of effective feedback and monitoring of each student’s scientific potential growth. Therefore, the research problem is what are the content, means and technologies for determining of teacher’s professional deficits in educational processes management of complex mathematics mastering based on the student’s personalization, visual and digital processes transformation of mathematical and computer modeling integration, mathematical, natural science, humanitarian and information cultures dialogue. The solution of this problem can give a powerful motivational charge to the study of mathematical disciplines and their applications based on modern scientific achievements adaptation; as a result, the interest in mathematics mastering will increase with real development of theoretical and empirical thinking (comparison, analogy, analysis, synthesis, etc.), mathematical literacy will be effectively formed and each student’s scientific potential and self-organization will be increased.

These tasks can be successfully solved only by a creative person in the conditions of difficult overcoming and enriched by cognitive activity in a saturated mathematical and information environment of mastering mathematics. Therefore, main mechanism for student’s creativity development and forming of student’s functional (mathematical) literacy in the modern period can be as teaching mathematics of complex knowledge, including a modern achievement in science. Thus, teaching mathematics at school should take place in an information-rich educational environment of complex level knowledge mastering in the context of mathematical, informational, humanitarian and natural science cultures dialogue and teacher’s and student’s didactic efforts integration in the direction of essences revealing of basic educational elements (concepts, theorems, procedures, algorithms, ideas) as a phenomenon in fundamental education. It is necessary to build the hierarchies of multi-level complex knowledge, methods and tools in cognitive activity, relying on didactic rules and patterns of mastering mathematical activity based on a synergetic approach. There is
a need to develop a distance learning environment for mathematical disciplines within the framework of developer’s methodological initiatives deployment – mathematics teachers, as well as a complexes of on-line courses and distance learning environments; it is necessary to be able to develop the provision of ICT support tools (including the mathematical package of computer algebra Mathematica, GeoGebra, Lego Mindstorms, Arduino, etc.) in complex systems development and tasks in mathematics teaching; using the "tetrad" technology in student’s research activities (V.S. Sekovanov [14], E.I. Smirnov [2]): student’s four types of creative activity to be performed: a) creative mathematical activity; b) construction of fractal sets with algorithms and high-level programming languages development; c) performing laboratory work in mathematics with computer experiments; d) learning of scientist’s creative biographies and an artistic compositions creating by fractals and ICT using. All teacher’s professional competencies characterize of complex knowledge synergy manifestation in mathematical education at school based on modern achievements in science adaptation, mainly in the forms of integrative and elective courses implementation, project activities and web quests, laboratory calculation and resource classes, including in gaming activities. Thus, pedagogical mastering support of complex systems and knowledge processes for students should be carried out by a teacher who has his own experience in complex mathematical knowledge mastering, aimed at student’s personal qualities developing, techniques and methods learning manifestation in non-standard meta-subject "problem areas" of student’s mathematical activity in a saturated information and educational environment.

Therefore, it is important to determine the modern teacher’s readiness to manage the student’s complex knowledge development (modern achievements in science), including determining the state of their subject and methodological deficits using Google Forms tools based on case-test methodology using.

**Methodology and technologies**

*The object of the study* is to determine the teacher's readiness to adapt the modern achievements in science to school mathematics and the ability to manage the process of student’s complex knowledge mastering. *The subject of the study* is the identification of teacher’s subject and methodological deficits by case-test diagnostics method using Google Forms tools.

Complexity is a diversity in unity. Problems solving of computational complexity (A. Turing, S. Cook, M. Rabin et al.) showed that it is the time characteristics that play the most important role in assessing the complexity of the problem (P-class problems (P-difficulty) – polynomial time, the traveling salesman problem – exponential time, etc.). There are the approaches when complexity is associated with the time of system formation or with its hierarchical structure, as well as with the probability of system formation from an initial element, sometimes complexity can mean the system ability to generate the semiotic information links and to interact with the external environment on their basis, allowing to implement a hierarchical management structure. The post-non-classical thinking of modern individual, based on nonlinear surrounding reality, situationality and uncertainty in decision-making, multiple goal-setting and choice ambiguity, strongly dictates the need and possibility of new knowledge mastering and accepting (mathematical literacy) by
overcoming the complex (modern achievements in science), including new knowledge, as an imperative of from chaos to order transition. Thus, a student's mathematical literacy can act as an attractor of phased deployment iterations of generalized procedures symbiosis learning (universal educational actions) and the processes of complex knowledge adaptation to basic educational elements development of school mathematics. This dictates the need to build, investigate and consider a multi-level complex as a condition for order parameters building and dynamically stable transition states a new level of complexity. Complex knowledge arises in complex systems and generates the multiple complex tasks of modern achievements in science. It is student’s complex knowledge development that makes it possible to create a situation leading to maintain ability of dynamic stability state (the formation of mathematical literacy) with acceptable values of internal or external disturbances (fluctuations) of mathematical activity in the processes of generalized constructs adaptation in modern achievements in science learning. Complex knowledge is the result of knowledge about the content complexity in semiotic and informational connections of nonlinear systems, objects and phenomena of real and virtual world, represented in the unity of descriptive and computational diversity and representation hierarchies [27]. At the same time, a personal-activity approach is necessarily implemented (L.S. Vygotsky, S.N. Dvoryatkina [10], V.W. Ushnitskaya [12] et al.), based on personality personification, training activity in subject and taking into account the preferences and features of personal development, actualization of emotional response to applied effect of knowledge and competencies being formed (A. Maslow, I.E. Unt, A. I. Kreyk [8] et al.).

One of the fundamental methodological ideas that form the teacher's readiness to accompany the mastering processes of student’s complex mathematical knowledge learning is synergetic approach as a basic mechanism of individual self-organization. Mathematical education synergy in this case will be considered by us as a symbiosis and a qualitative change in nonlinear effects of individual's self-organization and self-development during mathematical activity development in complex stochastic processes management based on different factors and principles coordination in three contexts: substantive (semiotic), procedural (imitation) and socio-adaptive [16]. The latter aspects are especially important in pedagogical systems due to the possibility of additional horizontal links establishing based on cultures dialogue implementation of A.A. Verbitsky's contextual approach (A.A. Verbitsky [1], S.I. Soroko [13], E.I. Smirnov [26]). Positive changes associated with the manifestation of mathematical education synergy generate the internal mechanisms deployment of student’s self-organization during mathematical constructs development at ever new complexity levels, while updating the forming ways of their mathematical literacy. An important circumstance is that cognitive processes acquire the integrative quality of enriching natural science knowledge with a humanitarian aspect, humanitarian knowledge acquires a scientific basis for substantiating the using ways of natural science and mathematical apparatus and methods.

In the process of complex knowledge generalized constructs learning, the actualization peculiarities of influence external factors are manifested in the method and forms of mathematical objects and procedures essence founding based on the multiplicity of goal-setting, stages and hierarchies construction of sign-symbolic and figurative-geometric activities (E.I. Smirnov [19], T. Uygun [25]), including a creative search and analysis of
problem’s side solutions to the information technologies using and network interactions, variability and parametrization of bifurcation transitions identifying and attraction basins in multi-stage mathematical and informational tasks based on ensuring coherence of flows information during cultures dialogue. The basic tool of personality-activity approach is visual modeling processes and student’s personal preferences individualization in the form of founding spirals and clusters deployment of personality experience – as an integral integrating mechanism for essential connections manifestation of complex knowledge generalized constructs and personality qualities formation. The integrity and orientation of this generalized construct is determined by blocks deployment of meaningful, motivational and applied components based on generic theoretical generalization and technological understanding construction its specific manifestations. At the same time, it is important to note that increasing complexity in open and non-equilibrium systems (such is mathematical education) it is not a destructive mechanism, but on the contrary creates the necessary paths and transitions to new level of development and self-development. The study of complexity as philosophical concept (G.V. Hegel, I. Prigozhin [11], H. Haken et al.) is confirmed by extensive experimental material, practice and the interdependence of integrative processes in science, technology, social transformation and economics. The attributes of polyvalence, multipolarity, emergence and disequilibrium of modern world manifest themselves in the form of regular transitions to higher levels of complexity in the context of concrete universal development theory (St. Beer, J. von Neumann et al.). It is noted that the difficulty in achieving certain critical levels is a synthetic characteristic of self-organization ability, the ability to develop and self-develop student’s thinking and personal qualities. Scientists, philosophers, teachers and psychologists (S.P. Kurdyumov, K. Mainzer, A.N. Poddiakov [9] et al.) have shown that effective personal development is possible when mastering concern with complex systems and knowledge (of different levels of its complexity), when creating situations of overcoming difficulties during knowledge development and world’s unified picture and scientific worldview formation. In complexity cognition the process of cognition itself "becomes a communication, a loop between cognition (phenomenon, object) and the cognition of this cognition" (E. Morin).

Such a mechanism can be the launch of "factor-impulse" of individual self-organization during complex knowledge development and adaptation (E.I. Smirnov [15]):

- by means of content updating of complex knowledge generalized constructs, including fractal structures as meaningful zones of bifurcation and integrity at increasingly complex levels of mathematical and computer modeling integration. At the same time, agreed stages of empirical (observation of individual manifestations and activity patterns of self-organization; identification of facts and their quantitative certainty; identification of structural, statistical, phenomenological laws; theory as an organized set of empirical laws) are deployed (G.V. Kalinina [3], S. Fatahi [23], R.R. Maaliw III [24]);
- by means of mastering actualization of generalized rules and values in visual-digital models of complex knowledge as attractors of personality’s founding of abilities development and understanding processes (E.I. Smirnov [16]);
- cultures dialogue and interdisciplinary integration as a means of integrative processes deploying, multiplicity of goal-setting and coherence of ways to search for truth,
emotional response to applied effects and awareness of an information availability and pedagogical support (S.A. Rozanova [29], E.I. Smirnov [20]).

School mathematics should be focused on modern achievements adapting in science (complex knowledge), creating an effective motivational field of learning, ability to "launch" a self-organization processes and improving the quality of mastering mathematics. An effective construct may be on following stages deployment in complex knowledge’s synergy manifestation in mathematical education at school. It will be as a mechanism for student’s mathematical literacy formation: motivational (self-actualization ("I'm interested in this")); approximate the information saturation (self-determination ("what can I do")); procedural-activity (self-organization ("I am able to manage the process")); control-correctional (evaluation of empirical verification of results); generalizing-transformative (self-development of personality ("I can do something new")); at the same time, it is necessary to develop the methods for selection, justification and development of psycho-diagnostic techniques and evaluation on procedures identification of teacher’s professional deficits and technologies for identifying of synergetic effects in mathematics teaching.

It is necessary to create the conditions for internal processes disclosure and an adequacy mechanisms of cognitive processes and external acts development based on the person identification in complex knowledge and procedures process mastering: personalization – subject representation in other people; isolation – an individual isolation in interaction with other people; an individual appropriation of comprehensive human essence (Z. Freud, Yu.B. Hippenreiter, A.V. Khutorskoy et al.).

Together with A.N. Poddiakov [9], we note the following features in complex problems solving, the implementation of which can actually lead to increase student’s mathematical literacy and creativity in mathematics mastering:

- in the behavior and development of a complex dynamic system, such as mathematical education, there is always a share of uncertainty and unpredictability; it requires a variety of different descriptions and solutions, both in content and in cognitive processes that differ from each other and complement each other; no less effective tools are the concepts of lax and fuzzy, constructed on the basis of empirical, not theoretical generalizations, study of which is impossible without the using of computer and mathematical modeling;
- complex system of educational elements mastering is characterized by changes not only at the level of specific manifestations, but also at the level of its essence (generalized constructs), which is most significant for updating of understanding processes and presence the self-organization of developing effects. In complex educational systems, effective rules (founding modes (E.I. Smirnov [15])) of an entity phased deployment can be distinguished (including of inverse problems method in self-organization theory (G.G. Malinetsky [4])), but they will inevitably be quite variable in self-organization types based on visual modeling implementation (E.I. Smirnov [18]) and fundamentally dependent of context;
- theoretical models of high arbitrarily levels are fundamentally limited. Various search samples are needed (experimental cross–sections, comparative analysis of specific manifestations, computer modeling, analogies, analysis through synthesis, etc.) for
effective research of complex dynamic systems - real interactions with the system, and not only theoretical activity with its abstract models [30];

• goal-setting variability is necessary when learning a complex system - setting diverse, diverse and multi-level goals (multiple goal-setting) that can compete with each other. One of the main person's emotional states in complex systems learning in mathematical education is uncertainty, doubt, willingness to accept a twofold (based on prediction and random) of actions results, etc.;

• results of human activity with complex system content and mathematical education methods, results of interaction with it cannot be predicted completely, exhaustively. Only probabilistically guaranteed educational outcomes are possible. Moreover, along with direct, predictable results of education and side variety of unpredictable products of personal development and mathematical activity are formed, both at school and at university.

This study uses the case-test diagnostics method of teacher's professional deficits of Google Forms tools using. The above components and characteristics of teacher's readiness to accompany students in complex systems and knowledge development determine the content and meters direction and composition of teacher's professional deficits. One of the modern means of personal qualities diagnosing are case tests (or test cases), namely, a high-level test case will be used – a test scenario with abstract preconditions, input data, expected results, postconditions and actions (E.B. Morgunov [6], O.V. Shvareva [21]). The means of diagnostic materials and results presenting was Google Forms, an online tool, one of the cloud services that allows you to create forms for data collection, online testing. This cross-platform service has attracted attention due to the possibility of variable responses, tables and videos using and high level of usability.

Testing methodology

*Entrance control:* A studying of teachers' competence deficits.

The purpose of studying: to study the level of proficiency in subject and methodological competencies of teaching staff, ensuring the formation of student’s mathematical and digital literacy in mastering process of complex systems and knowledge.

*Research objectives:*

- organization and conduct of teacher’s subject and methodological competencies research on developed basis of evaluation materials (Examples 1, 2);
- results analysis of teacher’s subject and methodological competencies;
- recommendations for teacher’s subject and methodological competencies development in the process of scientific and methodological support of teaching staff.

The participants are teachers of municipal educational organizations that carry out the educational activities according to educational programs of basic general education. Evaluation materials are developed in accordance with the following principles:

- compliance with the methodology and conceptual framework of PISA international comparative studies;
- focus on the labor functions and teacher’s specified labor actions in updated draft of teacher’s professional standard.
The main characteristics of measuring materials. Diagnostics is carried out in tests form of subject and methodological competencies online. The execution time is no more than 60 minutes.

Evaluation materials

of teachers' proficiency in measurement of subject competencies in the field of functional (mathematical) literacy (3 Standards and 9 tasks (distance; functional; revolving door) + 10 tasks of special competencies – for example, robotic devices) are presented (Fig.1, Table 2).

Preamble

The PISA International Test – a test that evaluates student’s functional literacy in different countries of the world and the ability to apply the knowledge in practice (test is conducted once every three years; teenagers aged 15 take part in it) – gives results that show that about a fifth of graduates of junior school worldwide do not reach the threshold level of functional literacy (in each area – mathematical, natural science and reading) and about a third of students – in one of the areas. Russia has been ranked from 27 to 35 in the world ranking in different years. The main difficulties in monitoring performing tasks of functional literacy formation are revealed: understanding the plot situation and translating it into the subject area language, finding a solution; working with information presented in various forms (figure, text, table, diagram); working with real data, quantities and units of measurement; interpreting the result taking into account the proposed situation; independence, the using of educational and life experience.

The purpose of the PISA study – to assess the student’s readiness to apply mathematics in everyday life – led to develop the special tools for identifying subject competencies. Students are offered not typical learning tasks of traditional monitoring studies of mathematical training, but close to real problem situations presented in some context and solved by means of mathematics available to the student. The context of the task is the features and elements of the environment presented in the text within the described situation. These situations are connected with various aspects of surrounding life and require more or less mathematization for their solution. The problems that are posed in these contexts are part of the experience or practice of students' participation in the real surrounding reality. As a result, PISA developers proposed three verbs to describe the student’s cognitive activity in solving problems: formulate, apply and interpret, which clearly reflect the main activities in solving life problems through the mathematics using. They point to three thought processes in which, as a rule, students will be involved with active participation in problems solving: to formulate a situation mathematically; to apply a mathematical concept, facts, thinking procedures; to interpret, use and evaluate a mathematical result. Similar problems arise for a teacher who not only has to know the principles, methods and techniques of solving and researching PISA - similar tasks, but also be able to develop the effective methods and technologies for solving practice-oriented tasks. This is the need for teacher’s knowledge, ability and competence in mathematical modeling methods using and mathematics formalism as an effective tool for individual socialization, solving and researching problems of society, technology, science and technology functioning. The proposed toolkit is designed
to identify the state and measure the level of teachers' proficiency in subject competencies in the field of student’s mathematical literacy formation when practice-oriented tasks solving.

Important types of practice-oriented tasks are optimization tasks – tasks in which one should choose the best (optimal) method from possible (acceptable) alternative options (methods) of actions. Such tasks are characterized by the presence of following three elements: 1) the set of possible actions (determined by task constraints), 2) search criterion (the parameter by which optimality is evaluated), 3) managed data (parameters that can be changed within the task constraints) and unmanaged data (parameters that are unchanged for the task being solved). In accordance with this, mathematical model of problem optimization describing the real situation contains the following elements: controlled variables – variables whose values are subject to change in the process of finding a solution to this problem; objective function (goal function) – a function that depends on controlled variables, which shows in what sense the solution should be the best; task constraints – a record of limiting or regulating conditions on controlled variables included in the task statement and not subject to change for this task; acceptable solution – a set of variable values that satisfy all task constraints.

**Example 1.** A tourist must climb to the top of a mountain, which has the shape of a regular octagonal pyramid – the angle at the top of each face is 30°, the length of the side edge is 1200 meters. The tourist must get from point A at the foot of the mountain to point O – to the top of the mountain, passing sequentially counterclockwise all the side faces, except the last one, where there is a waterfall. It is impossible to climb the slope of the mountain (the edges of the pyramid) at once, except for the eighth edge on which there is a lift to the top (point B) and BO = 300 meters. The power point 300 meters to the top on the 6th edge is point B5 (which must be reached) (Fig. 1 shows the pyramid unfolding). What is the length of the shortest path to the top, including the lift path? (Next, choose only one answer option A, B, C, D, E, F).

![Figure 1 Pyramid unfolding and possible path to the top](image)
A) Climbing from point A to point B, tourist must overcome 900 meters, if counted along the edge and if points A and B were on the same edge. The total number of edges to point B5 pass is 5. This means that we uniformly climb 180 meters with each next edge to B5, points B and B5 will be at the same height, then in a straight line to point B. Now summing the lengths of segments, we will find the length of entire polyline.

B) You can immediately climb to point B1 at a height of 900 meters along the adjacent edge (so that \( A_1 B_1 = 900 \)), and then go straight (in each face parallel to the corresponding edge of the base) (i.e., neither up nor down) to the lift. Then the shortest path will be equal to \( AB_1 + (1/4) * AA_1 * 6 + 300 \).

C) You can immediately climb to point B2 in a straight line on the sweep and to a height of 900 meters along the edge (so that \( A_2 B_2 = 900 \)), and then go straight to the lift all the time. Then the shortest path will be equal to \( AB_2 + (1/4) * AA_1 * 5 + 300 \).

D) It is necessary to follow the model: on the scan, connect points A and B5 with a straight line (where B5 is 300 meters away from the top), then – in a straight line to point B (in real ascent – first up to B6, then down to B). Then the shortest path will be equal to \( AB_5 + BBA_5 + 300 \).

E) It is necessary to follow the model: on the scan, connect points A and B5 with a straight line (where B5 is 300 meters away from the top), then straight to B6 (i.e., neither up nor down), then also to point B. Then the shortest path will be equal \( AB_5 + B_5 B_6 + B_6 B + 300 \).

F) You can immediately climb to point B3 in a straight line on the sweep and to a height of 900 meters along the edge, and then go straight to the lift all the time. Then the shortest path will be equal to \( AB_3 + (1/4) * AA_1 * 4 + 300 \).

**Evaluation materials**

Measuring the level of teachers' proficiency in *methodological competencies* in the field of functional (mathematical) literacy formation within the framework of specialized scientific research (Measuring the level of teachers' proficiency in competencies for assessing the quality of general education based on the practice of the international comparative study of PISA students' training. A tool for measuring a case test (Block 1-2 of 3 case tasks).

**Preamble**

In PISA study, it is believed that all types of mathematical activities that are highlighted at lower levels are an activities component inherent in a higher level compared to them. Fundamental mathematical abilities play a central role in determining the various levels of mathematical literacy success. Thus, in student’s mathematical literacy formation on the basis of complex knowledge mastering, the task is to create a rich information and educational environment for mathematics teaching by changing the educational programs content in the direction of complex knowledge mastering. This is realized in the course of step-by-step research and practice-oriented tasks solving and ability to effectively interpret tasks from real life: that is, to solve a wide range of tasks in various spheres of human activity, communication and social relations. Moreover, the task for the coming years is not only to achieve a stable threshold level in PISA testing, at which
students begin to demonstrate the application of knowledge and skills in simplest extracurricular situations. Priority is given to situations when student’s ability to use the existing knowledge and skills to obtain a new information is manifested, creative students who think independently and are able to function in difficult conditions and master of complex knowledge are required. This creates a precedent for expanding and deepening person’s experience based on his current state (it is necessary to take into account the student’s individual differences, i.e. practice-oriented tasks should be multi-level), motivational sphere formation and teaching development (due to the actualization of samples and adaptation of modern scientific knowledge and technologies that are in demand in life and accessible to perception), intellectual operations and abilities development based on founding mechanisms, mathematical and visual modeling of manifestation possibilities and correction of student’s functional, operational and instrumental competencies in mathematics complex constructs and procedures mastering. Thus, process implementation of functional literacy quality improving in mathematics development at school is now possible on the basis of synergetic principles and approaches actualization in context of modern achievements in science adapting to school mathematics. Such educational systems are characterized by the ability to fully meet the needs of each student in self-education and self-actualization when complex knowledge constructs mastering and set the value imperative of personal development. Therefore, mathematical, humanitarian, informational and natural science cultures dialogue is also necessary in mathematics development of complex knowledge, which activates the synergy mechanisms and is a factor of self-organization and a link in integral structures formation in mathematics teaching at school.

The work consists of 3 parts:
1. Information and statistical questionnaire (Block 1).
3. Measuring the level of teacher’s methodological competencies mastery in the field of functional (mathematical) literacy formation in mastering process. A tool for measuring is open PISA jobs https://fioco.ru/pisa or author’s PISA-like tasks developed by the research team (Block 3).

In parts 2-3, each respondent is given feedback in comment form containing recommendations for competence improving of measured element. The generalized results of competence assessment can be used to determine the individual development trajectories of teaching staff, including through recommendations for subject and methodological competencies development in the process of scientific and methodological support of teaching staff on the basis of the Center for Educational Technologies Transfer "New Didactics" in Yaroslavl State Pedagogical University.

Example 2. In your opinion, combination of which 3 following factors in mathematics teaching is most capable of effectively influencing on student’s functional (mathematical) literacy formation (Table 1)?
Table 1

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Creation of a rich information and educational environment and a motivational field of learning based on mathematical and computer modeling of complex knowledge in extracurricular and project-research activities</td>
</tr>
<tr>
<td>2</td>
<td>Provision and actualization of student’s individual educational trajectories development in main school on the basis of student’s self-organization and choice, comfort of mathematical activities development within the framework of Federal State Educational Standard</td>
</tr>
<tr>
<td>3</td>
<td>Level-based of complex knowledge adaptation in modern achievements in science (fractal geometry, theory of coding and encryption, fuzzy sets, etc.) to school mathematics by solving and researching of multi-stage practice-oriented tasks in extracurricular and project-research activities</td>
</tr>
<tr>
<td>4</td>
<td>Integration of mathematical, natural science, information and humanitarian knowledge, tools and activities in content mastering of elective courses in interdisciplinary orientation</td>
</tr>
<tr>
<td>5</td>
<td>The opportunity to systematically participate in mathematical Olympiads, competitions, projects and scientific lectures, including remotely, at school, regional and Russian levels</td>
</tr>
<tr>
<td>6</td>
<td>Digitalization of mathematical education based on small groups work and learning individualization through the introduction of adaptive, practice-oriented educational programs and textbooks in mathematics lessons, including remotely</td>
</tr>
<tr>
<td>7</td>
<td>Increasing the volume and time to master the PISA – practice-oriented complexes of similar tasks in mathematics lessons (or remotely) based on their analysis, research and level-based approach to mastering</td>
</tr>
</tbody>
</table>

Next, choose only one answer from options A, B, C, D, E, F:

A) 1, 3, 6  
B) 3, 4, 6  
C) 2, 5, 7  
D) 5, 6, 7  
E) 1, 4, 6  
F) 3, 6, 7

Example 3 Assessment case sheet of subject competencies

Table 2

<p>| The ability to extract, concretize, structure, highlight the main logical connections, plan implement; compare the result with the goal. Determine the dependence of conditions and results. Make decisions and make informed choices; mathematical model construction of real process or phenomenon; application of mathematical knowledge and methods to establish the connections in an applied problem | Option A | 0 | You’re wrong. Finding the most obvious way to solve a problem does not always correspond to the correct solution. If you connect points A and BS on the scan, then the path you specified will turn out to be a polyline, the length of which is greater than the length of the segment AB5. Further, the paths to point B may be equal and parallel to the edges of base, but your decision will still be wrong. Try to perform the additional constructions more carefully and check the adequacy of mathematical actions. |</p>
<table>
<thead>
<tr>
<th>Option</th>
<th>Score</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option B</td>
<td>0</td>
<td>You're wrong. The length of the polyline is always greater than the length of extreme points segment connecting. Therefore, moving in a straight line (neither up nor down) on the mountain, you form a polyline path, while polyline vertices will be on the edges of the pyramid – model of the mountain. You need to pay attention to the interpretation of movement real conditions on the mountain and actions on the model. This will help you in a future to successfully cope not only with tasks of this type, but also to develop methods of students teaching to solve PISA-like problems of an applied nature.</td>
</tr>
<tr>
<td>Option C</td>
<td>0</td>
<td>You're wrong. The length of the polyline is always greater than the length of extreme points segment connecting. Therefore, moving in a straight line (neither up nor down) on the mountain, you form a polyline path, while polyline vertices will be on the edges of the pyramid – model of the mountain. You need to pay attention to the interpretation of movement real conditions on the mountain and actions on the model. This will help you in a future to successfully cope not only with tasks of this type, but also to develop methods of students teaching to solve PISA-like problems of an applied nature.</td>
</tr>
<tr>
<td>Option D</td>
<td>2</td>
<td>You're right. The shortest distance between two points is the length of a straight-line segment, so mathematical actions on the sweep model led to similar results when climbing uphill. Well-known geometry theorems will allow us to find the length of this shortest path. In the future, try to develop methods of students teaching to solve PISA-like problems of an applied nature.</td>
</tr>
<tr>
<td>Option E</td>
<td>1</td>
<td>You're partially right. The movement from point B5 to B6 and from B6 to B in a straight line on the mountain (i.e., neither up nor down) will be longer than the movement from B5 to B in a straight line on the sweep model. You have correctly identified the movement from point A to point B5. Pay attention to the interpretation of real movement and action on the sweep model. This will help you in the future to successfully cope not only with tasks of this type, but also to develop methods of students teaching to solve PISA-like problems of an applied nature.</td>
</tr>
<tr>
<td>Option F</td>
<td>0</td>
<td>You're wrong. The length of the polyline is always greater than the length of the straight line connecting the extreme points. Therefore, moving in a straight line (neither up nor down) on the mountain, you form a polyline path, while the vertices of the polyline will be on the edges of the pyramid – model of the mountain. You need to pay attention to the interpretation of movement real conditions on the mountain and actions on the model. Try to perform additional constructions more carefully and check the adequacy of mathematical actions. This will help you in the future to successfully cope not only with tasks of this type, but also to develop methods of students teaching to solve PISA-like problems of an applied nature.</td>
</tr>
<tr>
<td>Absence of the task solution</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Results**

The integral complex of diagnostic materials implemented in Google Forms allows to reveal the teachers readiness to demonstrate and demonstrate the universal pedagogical competencies in complex mathematical knowledge mastering: possession, adaptation and transfer of modern achievements in science (including when practice-oriented tasks solving with student’s motivation and self-organization effects), the ability to form a rich information, educational and gaming environment by means of mathematical and computer modeling, willingness to adapt and knowledge using of science modern problems in solving practice-
oriented tasks in school mathematics teaching. Teachers of mathematics of the Perm Region (65 persons) with various experience of professional and pedagogical activity took part in the experiment (planned time of passing the test was 60 minutes). It was indicative the time that teachers spent on passing on 15 tasks of test: 9 – on subject competence; 6 – on methodological competence (Table 3):

<table>
<thead>
<tr>
<th>Time (mins)</th>
<th>0-30</th>
<th>30-60</th>
<th>60-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>32</td>
<td>22</td>
<td>13</td>
</tr>
</tbody>
</table>

This indicates that the majority of teachers (80%) is adequately assess their scientific and methodological potential, are tolerant of innovations in didactics, have their own experience in identifying "problem areas" of complex mathematical knowledge mastering (including the modern achievements in science). Nevertheless, about half of respondents perceived the diagnostics case-test form as an opportunity to intuitively a choose the right solution without thorough essence substantive analysis, methods and prediction of possible results of task solving (less than 30 minutes to complete the test). The sample mean “$x$” was 15.2 points, sample variance was 18.98, mean square deviation $sx$=4.36, representativeness error of sample average was 0.54. The accuracy of sample average determining, which is based on the formula $Cs = (Sx/x) \times 100\%$, was 3.56%. This means the sample representativeness, which is considered satisfactory if $Cs< 3-5\%$.

The subject competence of the teacher was determined by the results of case-test solution of 9 tasks in the presence of practice-oriented task standard solution with stages allocation, instructions and features of received solution interpretation. The first 3 tasks consisted of block 1 "Distance" (modeling were based on geometric formulas and algebraic actions), block 2 "Functional" (modeling were based on mutual transitions of sign systems) and block 3 "Revolving Door" (modeling were based on conscious choice, conditions and results interpretation). The results are presented in Table 4:

<table>
<thead>
<tr>
<th>Block/Result</th>
<th>Mean (scores / max)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34/134</td>
<td>25,4</td>
</tr>
<tr>
<td>2</td>
<td>72/134</td>
<td>54</td>
</tr>
<tr>
<td>3</td>
<td>64/134</td>
<td>47,8</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>56,7/134</td>
<td>43,7</td>
</tr>
</tbody>
</table>

Time intervals comparison of teacher’s test execution and low average results indicates the presence of subject deficits among mathematics teachers in the field of applying mathematical knowledge in practice-oriented tasks solving, as well as in adequate interpretation issues of task conditions and results. Therefore, professional mathematical training of future teacher should not only founding the content of school mathematics at generalized theoretical level (which is not always observed in curricula and curriculum), but
also reflect the content of modern achievements in science (fractal geometry, fuzzy sets and fuzzy logic, theory of encoding and encryption, cellular automata, etc.). The methods of adaptation approach to school mathematics will create an additional motivational and applied aspect based on mathematical and computer modeling integration.

Methodological competencies of teachers were determined in 6 tasks distributed in 2 blocks: Block 1 – Factors and problems of complex mathematical knowledge mastering; Block 2 – methodological possibilities of categories learning (space and form, change and dependencies, quantity, uncertainty and data) and play activities in mathematical education. The test results are presented in Table 5:

<table>
<thead>
<tr>
<th>Block/Result</th>
<th>Mean (scores / max)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>47,8</td>
</tr>
<tr>
<td>Σ</td>
<td>56,7/134</td>
<td>43,7</td>
</tr>
</tbody>
</table>

The latest results should be considered as positive – they indicate the teacher’s readiness for professional self-development, the ability to adapt and knowledge using of science modern problems in practice-oriented tasks solving with student’s motivation and self-organization effects, as well as the ability to form a rich information, educational and gaming environment by means of mathematical and computer modeling in practice-oriented tasks solving and organization of gaming activities. At the same time, subject deficits of teachers and factors hindering the processes of complex knowledge adapting to school mathematics were identified:

- difficulties in conceptual modeling of practice-oriented tasks, understanding of the plot situation and its translation into mathematics language;
- mutual transitions problems of sign systems in mathematical modeling;
- difficulties in working with real data, quantities and units of measurement.

Methodological deficits of teachers are manifested in the insufficiency of belief formation in personal development effectiveness by learning of complex level knowledge. Moreover, despite the fact that researchers (St. Beer, N. Wiener, J. von Neumann et al.) conclude that complexity is an integrating characteristic of self-organizing ability when certain critical levels of it are reached, the ability to effectively develop and self-develop student’s thinking and personal qualities. Teachers do not have enough knowledge of personality development laws and have difficulties in understanding and interpreting them. Thus, the resolution of contradictions related to the quality, mathematical literacy and success of student’s cognitive activity and objective complexity of knowledge is possible with increased of individualization process and student’s subjectivity development based on complex knowledge and procedures adaptation, as well as effective methods and techniques using for complex multilevel mathematical knowledge mastering in saturated information and educational environment. At the same time, the diagnostics showed teacher’s difficulties and professional deficits in generalizing the knowledge and motivation of students activities,
which as a consequence predicts the insufficient teacher’s ability to bring the student to
generalization level of concreate knowledge, to actualize a meta-subject competencies and
universal educational actions: without such an analysis and proper emphasis, student’s
knowledge and actions will be limited and will have a weak potential for expansion. One
of the ways to solve this problem is to study and adapt modern achievements in science to
school mathematics (fractal geometry, theory of encoding and encryption, fuzzy sets and
fuzzy logic, etc.).

Discussion

We agree with the authors (N. Meseșan [31], K.H. Perry [32], R. L. Long [33]) on the need
for the formation of student’s functional (mathematical) literacy through a comprehensive
solution of practice-oriented tasks of various directions (nature, society, infrastructure,
science, production). However, psychologists (A.N. Leontiev, N.G. Salmina) have proved that
efforts aimed only at the operationality of cognitive activity do not always lead to effective
results. It is necessary to internally organize the symbolic activity of the student with a high
level of educational motivation and reasonable structuring. Such an original direction is the
technology of complex knowledge study and teacher’s readiness determination to manage
the student’s cognitive activity in the development of complex knowledge generalized
constructs. The data obtained by us are consistent with the opinion of the authors (A.
Hašková [34], J. Wenner [35]) about the weak readiness of teachers to innovate using.

In PISA study it is believed that all types of mathematical activities that are highlighted
at lower levels are activities components inherent in a higher level compared to them.
It is revealed that the fundamental mathematical abilities of students play a central role
in determining the various levels of mathematical literacy success. Thus, in student’s
mathematical and digital literacy formation based on complex knowledge development, the
task of creating a rich information and educational environment for mathematics teaching
by changing the content and orientation of educational programs and practices (for example,
STEAM education) in the direction of complex knowledge mastering becomes a priority. This
should be realized in the course of step-by-step study of complex knowledge generalized
constructs and coordinated solution of practice-oriented tasks complexes and ability to
effectively interpret the tasks from real life: that is, to solve a wide range of tasks in various
spheres of modern areas in human activity, communication and social relations. The priority
for the teacher is situations when student’s ability to use the existing knowledge and skills
to obtain a new information is manifested, creative students who think independently and
are able to function in difficult conditions and master of complex knowledge are required.

Conclusion

Innovate activity with complex systems and knowledge creates a precedent for the
teacher’s readiness to expand and deepen the student’s experience based on his current state
(it is necessary to take into account the student’s individual differences, i.e. practice-oriented
tasks should be multi-level). Formation and development of motivational sphere of teaching
(due to the actualization of samples and adaptation of modern scientific knowledge and
technologies that are in demand in life and accessible to perception) develop of intellectual operations and abilities based on founding mechanisms, mathematical and visual modeling of manifestation possibilities and correction of student’s functional, operational and instrumental competencies and mathematical literacy in complex mathematical constructs and procedures mastering. The conducting research revealed an integral block of teacher’s subject and methodological deficits in complex systems and knowledge mastering and adapting to school mathematics and processes managing of student’s self-organization in research activities. Thus, process of functional (mathematical) literacy quality improving in mathematics development at school is now possible on the basis of synergetic principles actualization and approaches in the context of modern achievements in science adapting to school mathematics. Such educational systems are characterized by the ability to fully meet the needs of each student in self-education and self-actualization. It led to complex knowledge constructs mastering and set the value imperative of student’s personal development. Therefore, mathematical, informational, humanitarian and natural science cultures dialogue is also necessary in the development of complex mathematical knowledge, which activates the synergy mechanisms and being a self-organization factor and a link in integral structures formation in mathematics teaching at school.

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