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Реформа и контрреформа математического образования в СССР 1960-1970 годов

Введение. В 1960-70-х годах средняя школа в СССР оказалась вовлечена в мейнстрим международного движения за реформу математического образования. Изменению было подвержено как содержание математического образования, так и его идеология и структура. Сегодня необходим ретроспективный взгляд на исторические события 50-летней давности. Актуальность исследования определяется необходимостью переосмысления реформы математического образования 1960-70-х годов в новых условиях радикальных перемен и модернизации системы современного образования. Цель статьи – восстановить объективную историю проведения реформы и контрреформы математического образования в СССР, выявить предпосылки реформы, проанализировать причины ее провала и выявить положительные методические аспекты преобразований.

Методы исследования. Использовано более 100 источников, среди которых документы Архива Российской академии наук, учебники по математике (дореформенные, реформистские и пореформенные учебники), свидетельства участников реформы и контрреформы. Ведущими методологическими подходами выступили теоретико-философский, теоретико-математический и опытно-практический.

Результаты исследования. Анализ фактов позволил установить, что главными побудительными мотивами к проведению реформы в СССР были, с одной стороны, международное движение за обновление содержания школьного математического образования, опирающееся на идеи интернациональной группы математиков, именовавших себя Бурбаки, а с другой – чрезмерная вовлеченность академических структур в решение этого вопроса.

Обсуждение результатов исследования. Стремление высшего руководства СССР не отстать от тенденций, происходящих в науке, образовании и технике в мире сыграло негативную роль в области советского математического образования. Предоставление инициативы в выборе содержания математического образования «по западным лекалам» академическим работникам и структурам было ошибочным, поскольку не учитывалось мнение многотысячного сообщества учителей, практиков, методистов. Другой причиной неуспеха реформы авторы считают чрезмерную вовлеченность академических структур в ее реализацию.

Заключение. Исторический опыт показывает, что наибольших успехов в образовании и науке достигаются в периоды стабилизации. В математическом образовании России стабилизирующим фактором являются учебники, написанные в традициях систематического курса, уходящего истоками к «Началам» Евклида, представляющего собой некий культурно-исторический эталон. Один из главных выводов настоящего исследования состоит в том, что качество обучения тесно связано с сохранением отечественной педагогической традиции, ее фундаментальных принципов обучения (систеаматичности и последовательности).

Ключевые слова: реформа математического образования в СССР, теоретико-множественный подход, Международный математический конгресс, Бурбаки

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Reform and counter-reform of mathematics education in the USSR in 1960-1970

Introduction. In the 1960s and 1970s, a secondary school in the USSR became involved in the international mainstream for the reform of mathematics education. Both the content of mathematics education and its ideology and structure were subject to change. Today, the historical events of the last 50 years require a retrospective view. The relevance of the study is determined by the need to rethink the reform of mathematics education of the 1960s and 1970s in the new conditions of radical changes and modernization of the modern education system. The purpose of the study is to restore the objective history of the reform and counter-reform of mathematics education in the USSR, identify the prerequisites for the reform, analyze the reasons for its failure and identify the positive methodological aspects of transformations.

Research methods. More than 100 sources were used, including documents from the Archive of the Russian Academy of Sciences, textbooks on mathematics (pre-reform, reformist and post-reform textbooks), certificates of the participants of reform and counter-reforms. The leading methodological approaches included theoretical, philosophical, theoretical-mathematical and experimental-practical.

Research results. The analysis of the facts made it possible to establish that the main motives for the reform in the USSR were, on the one hand, the international movement for the new content of school mathematics education based on the ideas of an international group of mathematicians who called themselves Bourbaki, and on the other, the excessive involvement of academic structures (mathematicians and academicians) in solving this issue.

Results and discussion. The desire of the top leadership of the USSR not to lag behind the trends in science, education and technology in the world played a negative role in the field of Soviet mathematics education. The initiative in choosing the content of mathematics education “according to Western patterns” to academic workers and structures was erroneous, since the opinions of a community of thousands of teachers, practitioners, methodologists were not taken into account. According to the authors, another reason for the failure of the reform was the excessive involvement of academic structures.

Conclusion. Historical experience shows that the greatest successes in education and science are achieved during the periods of stabilization. In the mathematics education of Russia, the stabilizing factor is textbooks written in the traditions of a systematic course, going back to the Elements of Euclid, which is a kind of cultural and historical standard. One of the main conclusions of the study is that the quality of training is closely related to the preservation of the domestic pedagogical tradition, its fundamental principles of learning (systematics and consistency).

Keywords: reform of mathematics education in the USSR, theoretical and multiple approach, International Mathematical Congress, Bourbaki

For Reference:
In the 1950s, the ideas of French mathematicians acting under the collective pseudonym of Nicolas Bourbaki, the history of which was described by M.J. Barany [3] and O. Pekonen [20], gained great popularity in the world. The works of M. Sennechal [27] and V.I. Arnold [2] reveal the main idea of N. Bourbaki about building mathematics as a single science based on the concepts of set, transformation and structure. The implementation of this idea led to the formalization of mathematics, an increase in the role of its theoretical level and a weakening of the practical, and especially its geometric component. Bourbaki’s ideas were widely discussed at the International Congresses of Mathematicians (ICM) and were introduced into the teaching process in different countries.

A. Karp [12], R.A. Melnikov and O.A. Savvina [19] draw attention to the fact that the activities of International Mathematical Congresses were not limited to the framework of mathematical science, and discussed the issues of education. The study by A. Marmier shows the influence of the activities of the International Congresses on the reform of mathematics education in France [17].

The reconstruction of mathematics education based on formalization and theoretical and multiple ideology was the result of general trends in science in the first decades of the twentieth century. Thus, N.V. Bryanik writes: “In the first decades of the 20th century, researchers (in physics, mathematics, linguistics, literary criticism) came to the conclusion that it is the laws of statics/synchrony/stability that make it possible to reveal the inner world of the studied objects, their relative autonomy” [8]. The philosophical foundations of the modernization of education based on the new structure of scientific knowledge are considered in the works of V.L. Gapontsev, V.A. Fedorov, E.M. Dorozhkin [9], M.G. Gapontseva, A.N. Khuziakmetov, etc. [10].

A systematic (evidence-based) course of geometry was adopted in Russia in the late 19th – first half of the 20th century. At this time, the most famous were the mathematics textbooks by A.P. Kiselev, in which a successful balance was found between theoretical information from the Euclid’s book and practical material.


Reformist ideas began to be expressed in candidate theses already in the 1950s. The examples are the theses of M.A. Preobrazhensky “Elements of set theory in connection with the development of logical thinking of schoolchildren” (1951), V.M. Chistyakov “Movement to increase the theoretical level in teaching elementary geometry in the Russian school of the 19th and early 20th centuries” (1952). In the early 1960s, several master’s theses were defended, the authors of which were inspired by the ideas of the reform: E.L. Mokrushin “Development of the concept of infinity in the course of mathematics in grades 5-8 of a secondary school” (Moscow, 1961); G.L. Gukasyan “Development of the concept of functional dependence in the courses of arithmetic and secondary school algebra” (Yerevan, 1961); Yu.M. Kolyagin “On the issue of reforming mathematics education and a new formulation of teaching arithmetic in the Soviet school” (Moscow, 1963).
In the mid and late 1960s, the theoretical justification for the reform was designed to provide the candidate thesis by V.M. Bots “On the issue of modernizing the teaching of mathematics in grades 5-6” (Moscow, 1966); V.N. Shonia “Ideas of plurality and probability in the course of high school mathematics” (Tbilisi, 1967); N.M. Roganovsky “Axiomatic construction of a school course in stereometry with the involvement of ideas of geometric transformations” (Moscow, 1969).

Thus, by 1968, not only in the content of mathematics education was changed, but also its ideology and structure. According to V.A. Testov, the reform contributed to the development of algebra teaching at higher educational institutions [28]. Yu.M. Kolyagin believes that the mistake in the field of higher pedagogical education was the exclusion of elementary mathematics from the curriculum, which was previously studied for four years, and consisted of theoretical arithmetic, elementary algebra, elementary geometry and trigonometry (with the elements of spherical trigonometry) [13].

At the same time, the most painful for the school was the “restructuring” of the content of school geometry textbooks, which led to negative consequences. For example, M. Boyko [6; 7] and Yu.M. Kolyagin [13] draw attention to poor results of entrance tests.

There have been and continue to be various assessments of this landmark event in the history of mathematics education in the USSR, from enthusiastic (A.M. Abramov [1]) to categorically negative (V.S. Vladimirov, A.N. Tikhonov, L.S. Pontryagin [29]). This led to a clash of opinions in the scientific and pedagogical community. Mathematicians were divided into prominent supporters and opponents of the reform. The arguments of the opponents turned out to be more convincing marking the stage of the counter-reform began, which rather painfully affected the fate of the supporters of the reform. A detailed analysis of the origins of the reform and its consequences to this day remains out of the field of view of educational historians. A single example can be the study by B.T. Polyak [21], in which the author mentions this problem within the framework of the history of mathematical programming in the USSR.

More than 50 years have passed since those historical events, which today makes it possible to overestimate the past events in a more balanced and reasoned manner. S. Rezat, L. Fan, B. Pepin [25] propose to focus the mathematics education on changes in curricula. However, this approach does not reveal the origins of changes. To obtain a more objective historical picture, it is necessary to consider the social and political phenomena preceding and accompanying reforms of mathematics education.

The purpose of the study is to restore the history of the reform and counter-reform of mathematics education in the USSR, identify the prerequisites for the reform, analyze the causes of its failures and identify the positive methodological aspects of transformations.

Research methods

The study was conducted on the basis of the analysis of more than 100 sources. Since basis was the criterion for the objectivity of the presentation of historical events, preference was given to the analysis of primary sources. First, these were the documents of the Archive of the Russian Academy of Sciences (Fund 1860, Inventory 1, Case 83 “Minutes of the meeting of the Mathematics Department of the Russian Academy of Sciences”), materials of the XII, XIV, XV, XVI International Congresses of Mathematicians (ICM), materials of the XIX Congress of the Communist Party of the Soviet Union, regulatory
documents (mathematics programs and their projects (1968, 1978), recommendations by the Ministry of Education. A large group of sources included textbooks on mathematics, which, according to the chronological criterion, were divided into three groups: 1) pre-reform textbooks (textbooks by A.P. Kiselev, a textbook on geometry by N.N. Nikitin and A.I. Fetisov, a textbook on geometry by V.G. Boltyansky and I.M. Yaglom), 2) reformist textbooks (edited by A.N. Kolmogorov); 3) post-reform textbooks (edited by A.N. Tikhonov, a textbook on geometry by A.V. Pogorelov, etc.)

Second, these were the evidence of the participants of the reform and counter-reform recorded in written sources (publications by V.G. Boltyansky, N.Ya. Vilenkin, I.M. Yaglom, A.I. Markushevich, V.I. Levin, Yu.M. Kolyagin, L.S. Pontryagin and others, theses of V.M. Chistyakov, Yu.M. Kolyagin, etc.). Literature was also analyzed, which assesses the state of mathematics education in the world and in the USSR in the 1960s and 1980s: from the standpoint of the theoretical and philosophical approach (N.V. Bryanik, V.L. Gapontsev, V.A. Fedorov, E.M. Dorozhkin), theoretical and mathematical (A. Karp, A.M. Marmier, I.G. Malyshev), experimental practice (Yu.M. Kolyagin, I.P. Kostenko).

**Research results**

There are different opinions on what is considered the origins of the reform and the counter-reform that soon followed. Thus, the modern author I.P. Kostenko [14] believes that a significant role was played by the renewal of the composition of the mathematics department at the USSR Academy of Sciences, which took place in 1936. Witnesses and participants I.K. Andronov, Yu.M. Kolyagin [13] and others point to the international nature of the reform movement, associate its beginning with the intensification of the activities of the group of French mathematicians N. Bourbaki and the resumption of the work of the Mathematical Congresses. It is also worth noting two more important events that took place in the USSR:

1) defense by I.V. Arnold (1900-1948) of the first thesis in the USSR “Theoretical Arithmetic” (1941) for the degree of the Doctor of Pedagogical Sciences;

2) establishment of the Academy of Pedagogical Sciences (1943).

As A.A. Popov notes: “The post-war period is characterized by great creative activity of teachers. Various issues on the methodology of mathematics were discussed at pedagogical readings conducted by the Academy of Pedagogical Sciences since 1945” [23, p. 215]. Political attitudes and an increase in the activity of pedagogical efforts should be considered as social reasons for the reform of mathematics education.

On June 29, 1949, at a session of the Academy of Pedagogical Sciences of the RSFSR, A.I. Markushevich made a report “On increasing the ideological and theoretical level of teaching mathematics in secondary school”. The idea of the need for changes was run through all his speech: “The content of school programs and textbooks on basic subjects has stepped so far forward and is saturated with ideas of advanced Soviet science that the very idea of using an old gymnasium textbook on these subjects would seem absurd” [18].

In our opinion, the origins of the reform in the USSR should be chronologically sought in the second half of the 1950s, when the era of relative stability came to the country that had barely recovered from the devastation caused by the Great Patriotic War of 1941-1945. In 1952, the decisions of the XIX Congress of the Communist Party of the Soviet Union (CPSU) indicated two important provisions:
1) to strengthen polytechnic education;
2) to prepare for the transition to compulsory secondary school.

To solve the first problem it was supposed to connect the content of school mathematics as much as possible with the teaching of physics, chemistry and astronomy. For example, when studying such branches of physics as mechanics, geometric optics, the theory of the electrostatic and electromagnetic field, schoolchildren needed strong knowledge in geometry. In organic chemistry, when students dealt with complex molecules of a substance, it is important to have a clear idea of the spatial arrangement of atoms, i.e., knowledge in the field of stereometry. Without the knowledge of geometry, one cannot do well in geodesy, crystallography, construction (construction of buildings, bridges, aqueducts, laying roads) and even when cutting fabric, leather, sheets of metal or plywood, when an important geometric problem arises: cut out figures of a certain shape from an existing piece of material and, moreover, so that the number of pieces is minimal. Mathematical knowledge and real measuring skills are also needed in military affairs, for example, the gunner of an artillery gun needs to clearly maintain the angle at which the shell will be fired at the target.

The implementation of the guidelines of the XIX Congress of the CPSU was reflected in the strengthening of the practical focus in the teaching of educational subjects (including mathematics). Yu.M. Kolyagin draws attention to the fact that in the new program introduced in the 1953-1954 academic year, special importance was attached to students’ calculating and constructive skills, the ability to use tables, a logarithmic ruler; modeling, measurement works on the ground, etc. [13, p. 3]. As a result, in 1953-1954 there were changes related to the training of teachers. Many provincial teacher institutes (trained secondary school teachers with the right to teach in the first seven classes) received the status of pedagogical institutes (trained teachers for high schools), and physics and technology departments appeared at their faculties of physics and mathematics. This period for school mathematics education still remained stable.

It is important to note that in the 1950s, the international activities of mathematicians from different countries interrupted by the Second World War revived again. In the fall of 1954, the XII International Congress of Mathematicians was held in Amsterdam, within the framework of which the International Commission on Mathematics education resumed its activities. At one of the sections, a professor of mathematics from Yugoslavia Djuro Kurepa (1907-1993) made a presentation. The key point was that “modern mathematical ideas (sets, correspondences, relationships, structures) make it possible to consider mathematics more efficiently than before” and on the basis of this he came to the conclusion: “in this direction, mathematics education in school should be developed”, i.e., in the spirit of Bourbaki. It should be noted that this Congress was attended by A.N. Kolmogorov (1903-1987), who made a plenary report at its closure. According to the philosopher V.M. Reznikov, at that time A.N. Kolmogorov highly appreciated “good creative professional relations with French mathematicians” [26, p. 37].

In July 1956, the XIX International Conference on Public Education was held in Geneva (Switzerland). Its program included three items, one of which was devoted to the problems of teaching mathematics in secondary schools. The USSR was represented at this forum by Alexei Ivanovich Markushevich. A Swiss professor of psychology Jean Piaget (1896-1980) made a report related to mathematics education in secondary school. He raised the question that some general ideas of modern science (with reference to the ideas of N. Bourbaki) should be reflected in the construction of the course in mathematics in a secondary school.
A staunch supporter of the reconstruction of the teaching of mathematics in secondary school was the Professor of the University of London and the Secretary of the International Commission for Mathematics Education C. Gattegno (1911-1988). Let us turn to some of his thoughts, which resonated with the ideas of N. Bourbaki. In his opinion, it was time to travel the path from general to obtaining private, and not from private to generalization, as the predecessors believed.

So, the second encouraging reason for this reform, which swept the whole world, was the work of a group of French mathematicians united by a pseudonym Nicolas Bourbaki. The essence of their idea was the possibility of axiomatic construction of mathematics as a single science. In fact, these ideas were not new. Similar thoughts about the merger of mathematical disciplines into one subject were expressed by F. Klein (1849-1925) at the beginning of the twentieth century during the reform period.

Then there was the preparatory phase for the reform. The first sign testifying to the initial phase of the reform under the unspoken slogan “Euclid must leave” was the publication in 1956 of the experimental textbook “Geometry” for grades 6-9 of a seven-year and secondary school, approved by the Ministry of Education of the RSFSR. Its authors were Nikolai Nikiforovich Nikitin (1885-1966) – head of the mathematics teaching methodology sector of the Institute of Teaching Methods of the Academy of Pedagogical Sciences (APS), and A.I. Fetisov (1891-1979) – the first to defend his thesis on “Experience in teaching geometry in secondary school” (1946) at the APS Institute of Teaching Methods.

The authors set themselves the task of writing a textbook, firstly, different from the textbook A.P. Kiselev used for many years, and, secondly, the main idea of the textbook was to make a smooth transition from visual geometric representations to strengthening the formal-deductive component. However, existing facts indicate that in 1957 the author’s tandem broke up. It should be noted that the Nikitin-Fetisov textbook after two years of use left the foreground in teaching geometry, and was replaced by the textbook “Geometry: a textbook for seven-year and secondary schools (1959) written in the traditions of A.P. Kiselev. Only N.N. Nikitin remained its author, and A.I. Fetisov began work on his textbook.

In 1958, A.I. Markushevich became the Deputy Minister of Education of the RSFSR, which gave him new leverage to continue the reform, receiving support from the members of the secondary school section of the Moscow Mathematical Society. The members of this section V.G. Boltyansky, N.Ya. Vilenkin and I.M. Yaglom published an article in the journal “Mathematics Education” [5], containing proposals on the inclusion of the elements of mathematical analysis and vectors in the school curriculum, on restructuring the entire school course of mathematics on a functional basis, etc.

Gradually, the ideas of the reform gained more supporters. On the pages of the same journal, a famous mathematician, doctor of physical and mathematical sciences, Professor Viktor Iosifovich Levin (1909-1986), who for many years headed the Department of Mathematical Physics of the Moscow State Pedagogical Institute, appealed for updating mathematics education. The scientist argued: “If the archaism of some of the material of the modern program of our school is gradually eliminated as a concession to new trends, then the imperfection of methods of teaching mathematics is still relevant and greatly inhibits its reorganization” [15, pp. 145-146].

In 1957, a satellite was launched in the USSR. In the USA and a number of European countries – the countries of the First World, this event served as an occasion to stimulate discussion of the teaching of mathematics. In 1959, in Royaumont (France, Royaumont monastery in Asnières-sur-Oise), an international seminar was held on the
topic “New thinking in school mathematics”, which was attended by a limited number of representatives of several countries (Austria, Belgium, Iceland, France, etc.). One of the outstanding speeches was the report of Jean Dieudonné (1906-1992), calling for change with the words: “Down with Euclid! Death to triangles!”. Despite the fact that not everyone agreed with J. Dieudonné, the seminar nevertheless reached a consensus on the basic principles of building new mathematics and their introduction into school teaching. Therefore, K. Bjarnadóttir considers the conference as the starting point of the reform [4].

At the XIV International Mathematical Congress in Stockholm in 1962, it was stated that in a large number of Western countries the school course of mathematics teaches the elements of set theory and mathematical logic, the concept of modern algebra (groups, rings, fields, vectors), the beginning of probability theory and mathematical statistics. The desirability of modernizing mathematical terminology and symbolism was noted; it was proposed to exclude a number of traditional sections of the mathematics course from the school course (elementary geometry and trigonometry, squeeze arithmetic). The recommendations of the International Session on the Teaching of Mathematics at School, held in Athens in 1963, explicitly stated that “the basis of the school course in mathematics is the concepts of set, relationship, function”, noted “the need to have the idea of mathematical structures as an ideological thread of teaching” [11, p. 80].

For a short time (from 1962 to 1967) schools in the USSR shifted from 10-year to 11-year study period. At this time, a new attempt was made to modernize school mathematics textbooks. In 1962, an open competition was announced to create new mathematics textbooks. Professor B.V. Gnedenko became the chairman of the jury, professor V.I. Levin received the post of the chairman of the commission on arithmetic, professor of Moscow State University named after M.V. Lomonosov A.G. Kurosh and professor at the Moscow Aviation Institute N.F. Chetverukhin – in geometry.

It is worth noting that the issue of including the elements of probability theory in the mandatory course of school mathematics was also considered, this idea was actively promoted by A.N. Kolmogorov, B.V. Gnedenko, A.Ya. Khinchin and others. In the same years, N.Ya. Vilenkin wrote a number of textbooks on combinatorial analysis, in which the author considered numerous problems with immersion in the “life” plot of different levels of complexity. As a result, the opinions of the teachers were divided. Some considered it necessary to make a separate course of the foundations of probability theory, while the others supported the common combinatorial-probabilistic line. The antagonistic approaches were combined by the fact that all authors were dominated by the probabilistic material, and the elements of statistics were present in a very limited amount. As a result, the probabilistic material did not find a place in the final draft of the school mathematics program. This decision was made by A.N. Kolmogorov himself, who stated that there was no consensus in resolving the issue of introducing the first concepts of the probability theory into the mathematics of a secondary school.

In 1963, the textbook “Geometry” (for grade 9 of the secondary school) was published by V.G. Boltyansky and I.M. Yaglom. The book was recommended for publication by the Ministry of Education of the RSFSR. It contained two sections “Geometric transformations” and “Vector algebra” with tasks and exercises. Boltyansky-Yaglom’s textbook survived only two years. For the active supporters of the reform of mathematics education this fact meant that the reform was a serious matter, intolerant to haste, but requiring systematic and painstaking work.
The book “Geometry. Textbook on the high school program” by A.I. Fetisova was published in 1963. This book can be considered a reincarnation of part of the Nikitin-Fetisov textbook published in 1956. As before, the author relied on the idea of geometric transformations (central and axial symmetries, translation, rotation, similarity), he used them when proving all statements. Among the innovations we should note the section related to the theory of parallel projection and to constructions on a projection drawing. Besides, the term “vector” appears in the presentation of the metric part of the content. This book was already addressed to mathematics teachers and recommended for organizing club activities.

It should be noted that the idea of introducing geometric transformations into the school course of mathematics was later embodied in a counter-reform textbook by Professor A.V. Pogorelov (the textbook began to be massively used since 1982). A.V. Pogorelov managed to withstand a reasonable measure of the amount of material devoted to geometric transformations, which were not rejected either by the teaching community or students.

The recommendations of the International Session on the Teaching of Mathematics at School, held in Athens in 1963, explicitly stated that “the basis of the school course in mathematics is the concepts of set, relationship, function” and noted “the need to have the idea of mathematical structures, like the ideological thread of teaching”.

In 1964, the Commission on the Reform of Secondary Education was created at the USSR Academy of Sciences and the USSR Academy of Pedagogical Sciences. Its mathematical section was headed by the former Deputy Minister of Education of the USSR A.I. Markushevich and academician A.N. Kolmogorov. Only Professor I.K. Andronov joined the Commission from the Academy of Pedagogical Sciences.

The first document of the reform “The Scope of Mathematics for an Eight-Year School” appeared in the 1965 journal “Mathematics at School”. It was prepared by a group of members of the Commission on Mathematics Education of the Mathematical Department of the USSR Academy of Sciences (I.M. Gelfand, A.N. Kolmogorov, A.I. Markushevich, A.D. Myshkis, D.K. Faddeev, I.M. Yaglom).

In 1966, there was an impact on school mathematics education – trigonometry ceased to exist as an independent educational subject with its own separate textbook. I.G. Malyshev focuses his attention on this innovation: “It is worth paying attention to an important innovation – the elimination of trigonometry as a separate subject. It was a serious undermining of the level of mathematics education” [16, p. 20].

In 1966, the XV International Mathematical Congress was held in Moscow. One of the sections of the XV Congress was devoted to the reform of school mathematics education. The Congress inspired the supporters of the reform, new programs were soon compiled and new textbooks were written and immediately introduced in schools. A brochure was published with the text of a mathematics program with a circulation of 4,000 copies, which was sent to all major cities of the USSR. Methodologists were involved to finalize the program (until 1968): the head of the Mathematics Training Laboratory of the Research Institute of Content and Training Methods of the Academy of Sciences of the USSR G.G. Maslov, as well as the employees of this laboratory Yu.N. Makarychev, K.I. Neshkova, A.D. Semushin, A.A. Shershhevsky (mathematics teacher).

Under the new programs approved and implemented in 1968, radical changes were made to the content of school education in mathematics. The arithmetic course for grades 5-6 was replaced by a mathematics course. The course of algebra of the main school was proposed to be “penetrated” with the idea of the set, correspondence and function. The course of planimetry proposed to strengthen the idea of geometric transformations, to consider
the geometric figure as a set of points; increase the severity when considering geometric quantities; study the elements of vector calculus. The course of algebra and the elements of analysis in high school was proposed to be presented in the “epsilon-delta” language considering the concepts of the limit of a derivative, an antiderivative, a definite integral and a differential equation. A course of stereometry – to build, if possible, on a vector basis; at the end of the mathematics course – consider the system of axiomatic geometry construction.

Neither teaching nor teacher training institutes, nor pedagogical institutes, nor local education bodies were prepared for such a drastic change in the content and methods of teaching mathematics at schools. As Yu.M. Kolyagin testifies: “The situation was complicated by the fact that the authors of the new textbooks themselves, as well as the leadership of the Ministry of Education, were inconsistent in their program and methodological ideas” [13].

The fact that when the study of specific functions immediately began (for example, a linear function), schoolchildren dealt with neither discrete finite sets, but with continuous infinite sets, did not bother anyone. Some teachers noted that the introduced definition of a function “works” nowhere in the course of algebra, but this was considered a slight disadvantage.

Besides, there was a “pedagogical fork” between teaching mathematics and teaching physics. In mathematics lessons schoolchildren talked about a function as a correspondence, and in physics lessons the same schoolchildren talked about it as a dependent variable (and this was not the only “split”)” [13, p. 198].

Almost immediately after the school’s activity, the complete unsuitability of the new geometry textbook was revealed in a new way, which, by the order of the Minister of Education was followed by the abolition of traditional oral exams on geometry in grade 9.

Teachers with great difficulties learned the new program. Many experienced teachers left the school.

The negative consequences of innovations were sharply discovered when school graduates began to enter universities. It turned out that the applicants of even such prestigious universities as Moscow State University, Moscow Engineering Physics Institute, Moscow Institute of Physics and Technology cannot solve the square equation, do not know how to perform the simplest oral calculations, formally know the definitions of leading mathematical concepts, often not understanding their meaning.

The ideas of neo-reformers were actively introduced into the educational practice not only in the USSR, but also in a number of other countries. So, in the 1960s, experimental training began in Belgium under a program developed by G. Papi, in Italy – at the school of the teacher-mathematician E. Castelnuovo. In France, new programs were introduced in 1969. However, by 1972, the spirit of modern mathematics (Bourbaki’s ideas) was criticized. A. Marmier testifies that “Criticism was no longer limited to a few detractors, but began to spread to scholars, the public and the press. From all sides there was a condemnation of excessive abstract, cumbersome new programs and dogmatism” [17]. Some members of the APMEP (Association des professeurs de mathématiques de L’enseignement public) were against the reform as it was implemented. Henry Bareill, the association’s president and reform advocate, called for a “pause”. In 1974, reform pioneers Pierre Samuel and J. Diedonnet began to express doubts about the advisability of teaching modern mathematics. According to A. Marmier, criticism most affected “abstract language, which in itself, introduced by pseudo-concrete actions, sometimes reached the point of absurdity. The effect of discriminatory selection based on expressive abilities was doubled, scientific content was not learned, and teachers and students were left behind” [17].
A negative assessment of the Bourbaki’s ideas into school mathematics was expressed by the Polish mathematician G. Steinhaus (1887-1972). His main idea was that this “secular order” for many years have been building the foundation under the already finished building of modern mathematics with amazement-worthy endurance and consistency. I.G. Malysev explains: “This phrase is not a stylistic oversight: the building stands, and the foundation is laid under it” [16, p. 21].

In the report “Development of the mathematical activity of students and the role of problems in this development” made by the professor of Krakow Pedagogical Institute S. Krygovskaya on behalf of the International Commission for Mathematical Learning (ICMI) on the basis of the submitted materials of 9 sub-commissions (England, USA, Norway, Sweden, Germany, Belgium, Hungary, Poland) sounds: “...The axiomatic construction of the modern course of school mathematics is not consistent with a free search for topics and techniques of students. <...> The expulsion of classical Euclidean geometry from the school led to the exclusion of traditional structural problems, which led to emptiness”.

In 1972, the Second International Congress on Mathematics Education (ICME) was held in the British city of Exeter. French mathematician Rene Frederic Tom (1923-2002), who later became a member of the French Academy of Sciences, criticized modernism in teaching school mathematics. Famous mathematicians Rolf Nevanlinna (1895-1980) and Jean Leray (1906-1998), mathematician-teacher Hans Freudenthal (1905-1990), as well as other scientists also condemned the theoretical and multiple excesses.

All this indicates that over time the world pedagogical community realized the negative consequences of innovations in the school mathematics program. The reform in many countries was reorganized and then quickly abolished. There were many obstacles: haste on the part of the ministries of education, corporatism, conservatism of the “guardians of the temple of Euclid”, “ignorance of mathematicians and psychologists (supporters of the reform) regarding the real conditions of secondary education” [17].

Another mistake of the reformers, which is rarely pointed out, was the rejection of the practice of using a textbook and a problem book together in teaching school geometry (the problem book of N.A. Rybkin was adopted at school for decades, and it was it that even more than the textbook of A.P. Kiselyov set the tradition, determined the culture of the teacher). After several years, A.M. Abramov, a student, a faithful follower and co-author of academician A.N. Kolmogorov, writes: “...it seems a mistake to connect a textbook with a problem book in one book. On the one hand, this limits the teacher’s freedom by tying him to a textbook-specified task list. On the other hand, the authors of textbooks are doomed to search for each small topic of the “optimal” exercise system – an insoluble task” [1, p. 17].

In May 1978, there was a discussion of certain key issues related to school mathematics at a special meeting of the bureau of the Department of Mathematics (chaired by the President of the USSR Academy of Sciences M.V. Keldysh). A.N. Kolmogorov was invited. As a result, a critical resolution was adopted, and a special meeting of the Mathematics Department of the USSR Academy of Sciences was recommended, devoted entirely to the problems of the school. The meeting took place on December 5, 1978. L.S. Pontryagin (1908-1988) and A.N. Tikhonov (1906-1993) spoke most sharply about the reform. The ideas of the reform were supported only by two participants in the meeting – L.V. Kantorovich and S.L. Sobolev. As a result, the existing situation with school programs and textbooks was recognized as unsatisfactory, it was also proposed to the Ministry of Education of the RSFSR to create a new draft program in mathematics, while announcing
an open competition for writing experimental textbooks. Thus, firstly, a Commission on Mathematics Education was created consisting of academicians A.N. Tikhonov, I.M. Vinogradov (1891-1983), A.V. Pogorelov and L.S. Pontryagin, and secondly, the idea of the Ministry of Education of the RSFSR was supported to develop a new program, textbooks and conduct an experiment. Active work has begun to eliminate the negative consequences of the reform.

L.S. Pontryagin played a significant role both in shaping public opinion and in gaining support of the “counter-reformers” at the top. Two papers were published by researchers – “Ethics and Arithmetic” (“Socialist Industry”, May 26, 1979); “On mathematics and the quality of its teaching”, published in the main ideological journal of the Central Committee of the CPSU “Communist” (1980, No. 14) [22].

At a session of the Supreme Soviet of the USSR, the rector of Moscow State University, academician physicist A.A. Logunov (1926-2015) drew attention to the problems of school mathematics, and noted excessive abstractness, isolation from real images and needs and the scientism of school mathematics. In his opinion, the corresponding “masterpieces-textbooks” are able to completely kill not only the interest in mathematics, but also in the exact sciences in general.

It should be admitted that some mathematicians-teachers, who were enthusiastic about its implementation at the beginning of the reform, in those years adjusted their views on the construction of school mathematics. Thus, A.A. Stolyar, a well-known methodologist who worked at the Pedagogical Institute in Mogilev wrote: “The consistent use of a theoretical and multiple approach to building a school course in mathematics leads to a significant complication and is didactically unjustified by anything. This is undeniably confirmed experimentally and theoretically”. On the other hand, he noted, “the other extreme – the exclusion of all theoretical and multiple concepts and designations from school mathematics is also didactically unjustified” [30].

Many scientists called the restoration stage as the stage of returning to A.P. Kiselyov. As a result of active criticism, the geometry textbooks (edited by A.N. Kolmogorov and Z.A. Skopets) were excluded from the school program. The only geometry tutorials that could be used at schools at that time were two books by A.V. Pogorelov on elementary geometry (Planimetry and Stereometry) published in the 1970s. The forced decision to introduce the textbook “Geometry” by A.V. Pogorelov was made in the Central Committee of the CPSU.

Academician A.D. Alexandrov, who replaced A.N. Kolmogorov in 1980 as the chairman of the commission of the Educational and Methodological Council, headed the team that created textbooks on geometry. His co-authors were Professor of the Leningrad Pedagogical Institute A.L. Verner and one of the Leningrad teachers V.I. Ryzhik. It should also be noted that back in 1978, by the order of the Ministry of Education of the RSFSR, A.N. Tikhonov created two teams to write textbooks on algebra and geometry. The team of authors in algebra included Professor Sh.A. Alimov, Professor V.A. Ilyin; Professor Yu.V. Sidorov, Professor M.I. Shabunin; Doctor of Pedagogical Sciences, Deputy Director of the Research Institute of Schools of the Ministry of Education of the RSFSR Yu. M. Kolyagin. The team on geometry included Professor E.G. Poznyak, Professor V.F. Butuzov, Professor L.S. Atanasyan, Senior Researcher at the Physics Department of Moscow State University S.B. Kadomtsev. These textbooks turned out to be so successful that they continue to be used at the present time.
The significance of the newly obtained data on the origins of the reform and its failures for science is that the teaching of mathematics should be based on traditions. We agree with A.M. Marmier [17], O. Pekonen [20] that mathematics education in the 1960s in the world, including the USSR, was greatly influenced by the ideas of Nikolas Bourbaki. However, this is not consistent with the ideas of A.M. Abramov (exaggerating the role of A.N. Kolmogorov and diminishing the influence of Bourbaki and A.I. Markushevich) [1] and I.P. Kostenko (who saw the reason for the reform in updating the composition of the mathematics department at the USSR Academy of Sciences in 1936) [14] on the driving forces of the reform of mathematics education in the USSR. Besides, we believe that the search for ways to increase the scientific level of teaching school mathematics in the USSR was stimulated by social factors: a course towards polytechnization of education and an increase in the duration of secondary education.

The data obtained by the authors on the failures of the reform of mathematics education in the USSR and ways to correct them are fully consistent with the opinion of Yu.M. Kolyagin [13] and partially with Yu.P. Zolotukhin, who notes: “Its failure in the conditions of a single comprehensive school was due to many reasons, in particular, the lack of differentiation in training and variable programs, the imperfection of new textbooks, conservatism and the known unprepared teaching staff, insufficient methodological support” [30, p. 100].

In our opinion, methodological support for the reform was quite well developed. More than 10 theses of Soviet mathematicians devoted to the theoretical justification of the reform and several textbooks implementing new ideas in the field of arithmetic, geometry and algebra were revealed during the study. According to the authors, the key to the success of studying mathematics in the USSR is that it followed the traditions of building a course according to the canons laid down by Euclid. The rejection of these canons served as the main factor in the failure of the reform.

Another reason for the failure of the reform is the excessive involvement of academic structures in its implementation (mathematical theorists from the USSR Academy of Sciences). Since, assuming that the reform would be carried out not in a mass school, but in specialized schools (for example, in specialized boarding schools for gifted children or in schools (classes) with in-depth study of mathematics), the result would obviously be different: reform ideas were later quite successfully implemented in educational literature for specialized mathematical classes.

In the 1970s, several countries were covered by the movement for the reform of mathematics education (USSR, France, Belgium, Poland, etc.). For the first time, two groups of prerequisites for the reform in the USSR were identified: external and internal. On the one hand, the external motive for this movement was the work of a group of French mathematicians Bourbaki, who proposed to rebuild the course of mathematics based on the concepts of set, structure and transformation. On the other hand, at that time the USSR set the tasks of polytechnizing education and increasing the duration of secondary education, which stimulated the search for ways to increase the scientific level of teaching school
mathematics and involve school education scientists who were engaged in pure academic science and did not have practical teaching experience at schools.

Replacing the traditional systematic course of geometry with a course at a high theoretical level did not live up to the hopes of innovators in mathematics education, the reform was discontinued. At the same time, the reform ideas turned out to be more stable in the field of teaching algebra and the beginning of analysis. After revision, reform textbooks on algebra and mathematical analysis were used for a long time in schools, and the proposals made at that time on the inclusion of the elements of the probability theory in the school course of mathematics were returned at the beginning of the 21st century.

For the first time it is shown that the disadvantages of the reform in the USSR and other countries turned out to be similar: this is the formalization and excessive abstraction of the theoretical and multiple ideology, the cumbersome nature of new programs and textbooks. We believe that these disadvantages have not been noticed for a long time, because the scientific authority of the supporters of the reform was quite high.

The 1980s were characterized by the desire to overcome the mistakes of the Soviet school of the past. The difficult reformist textbook of geometry by A.N. Kolmogorov and others was replaced by the textbook of academician A.V. Pogorelov, who, although he represented traditional Euclidean geometry, was still written in a rather complex scientific language. Textbooks on algebra by Sh.A. Alimov and others and textbooks on geometry by L.S. Atanasyan and others were created under the guidance of A.N. Tikhonov. New textbooks in other mathematical disciplines began to appear, and the school began to gradually return the lost positive traditions.

Political events in the country (the collapse of the USSR in 1991, etc.) caused a new wave of education reforms, which continues to this day. This wave, unlike the reforms of the 1970s, is characterized by the second extreme – the imposition of a visual, practical course in the secondary school, which means a rollback of centuries and millennia ago, to pre-Euclidean prescription mathematics and ancient Indian contemplation. This sad trend was outlined during the modernization of education in Russia at the end of the 20th century.

However, historical experience shows that the greatest advances in education and science are achieved during the periods of stabilization. In the mathematics education of Russia, the stabilizing factor is textbooks written in the traditions of a systematic course, going back to the Elements of Euclid. The attempts to refuse teaching from Euclid end with a return to the methodological traditions of Euclid. One of the main conclusions of the study is that the quality of training is closely related to the preservation of the domestic pedagogical tradition, its fundamental principles of learning (systematics and consistency), which cannot be interrupted.

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